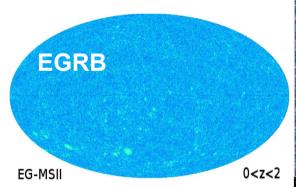
# Investigating the nature of dark matter with numerical simulations of structure formation



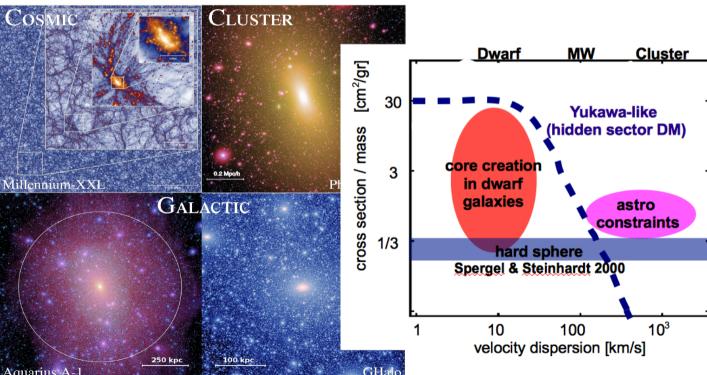


Jesús Zavala Franco (Marie Curie Fellow)









#### **Outline**

- Towards an effective theory of structure formation
  - Motivation: DM interactions and structure formation
  - Implementation of new DM physics in N-body simulations (seminar tomorrow)
  - Consequences for galaxy formation/evolution (seminar tomorrow)
- Using N-body simulations to obtain predictions for non-gravitational DM signals
  - Resolved phase-space structure (what we know)
  - Unresolved phase-space structure (extrapolation)

# The "standard model" of structure formation

The current model of structure formation is the Cold Dark Matter (CDM) model

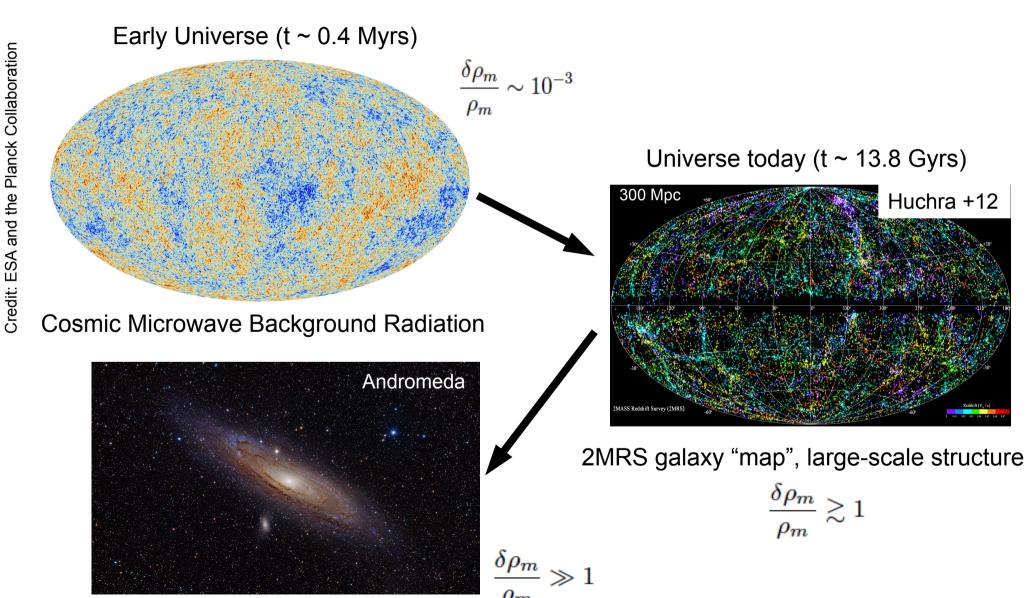
Hypothesis: DM is a new cold and collisionless particle

Galaxies form in a purely gravitational DM background, i.e., the nature of DM as a particle is irrelevant for galaxy formation and evolution

CDM is by itself an incomplete DM theory that needs completion from a particle physics model (all beyond SM: "exotic")

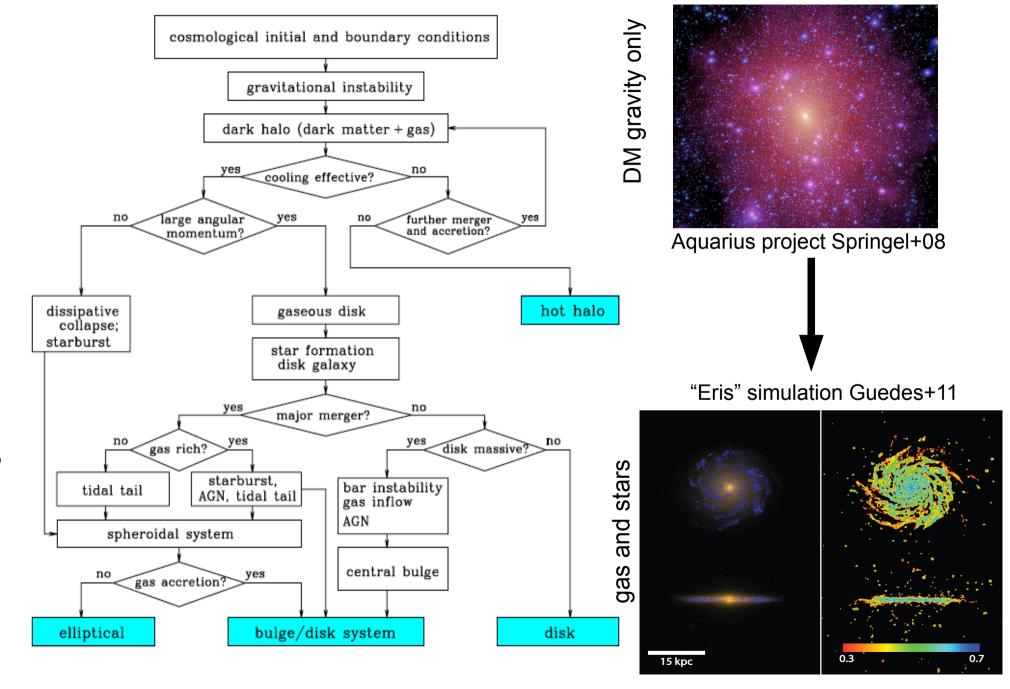
### **Dark Matter astrophysics**

The particle DM hypothesis is the cornerstone of the current theory of the formation and evolution of galaxies



galactic scales

#### Galaxy formation in a DM background



### Towards an effective theory of structure formation:

### motivation for additional DM physics in structure formation

#### A few remarks

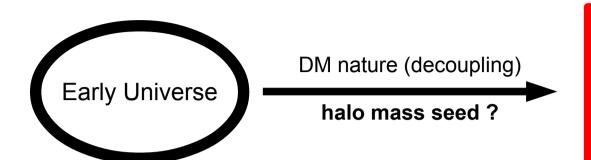
In CDM, galaxies form in a purely gravitational DM background, i.e., the nature of DM as a particle is irrelevant for galaxy formation and evolution

There is however, **no strong evidence** to support this **strong** hypothesis

A less stringent hypothesis preserves the success of CDM at large scales and predicts a distinct DM phase-space structure at smaller scales

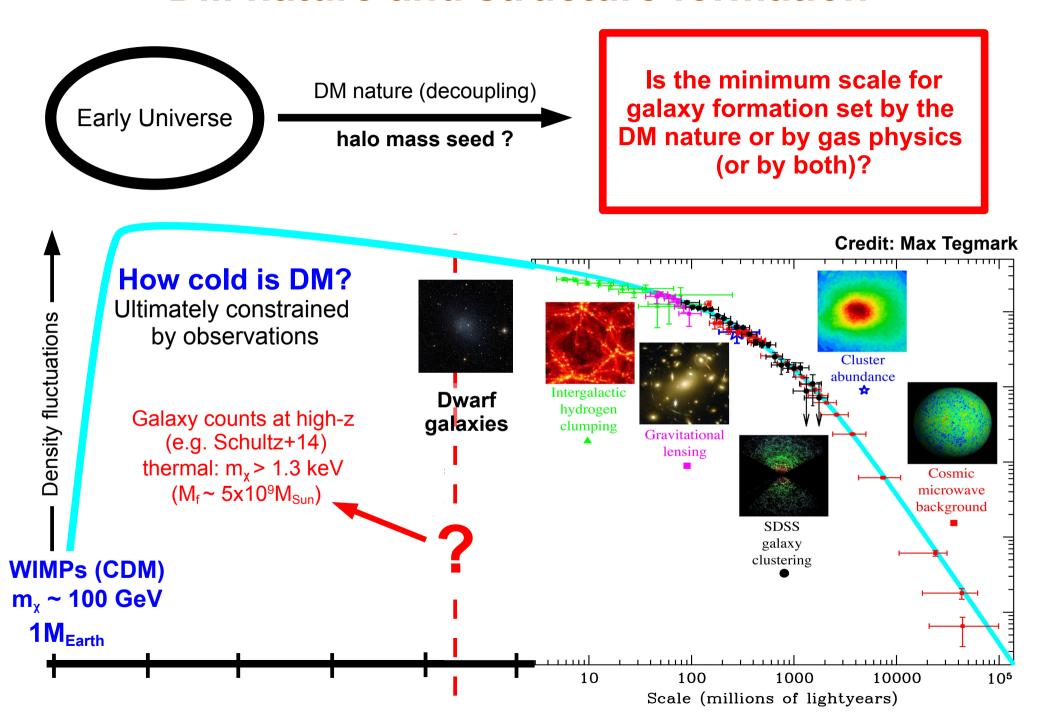
Although there is no indisputable evidence that the CDM model is wrong, there are reasonable physical motivations to consider alternatives

#### **DM** nature and structure formation



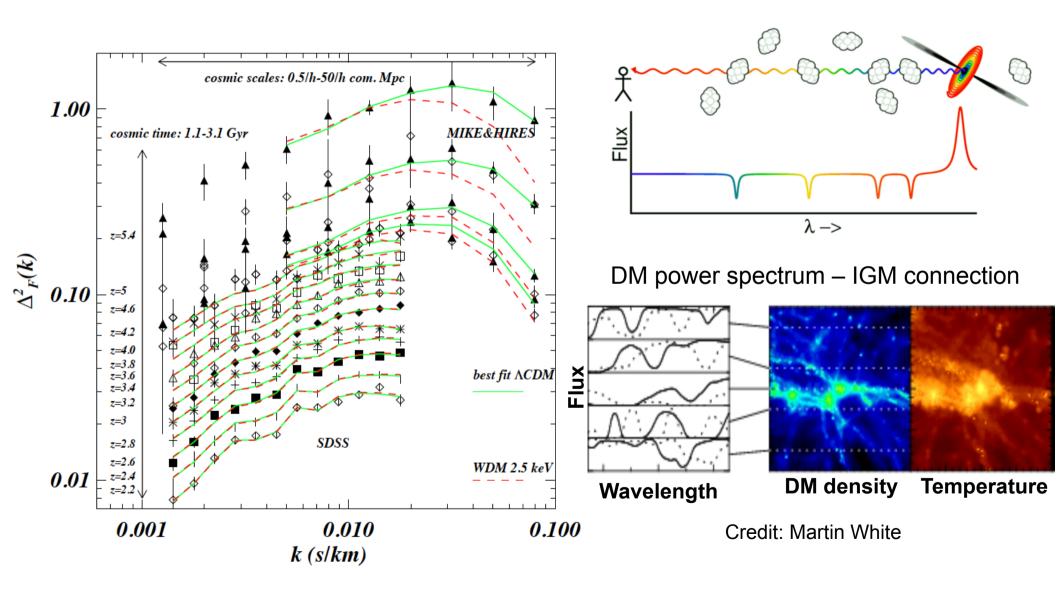
Is the minimum scale for galaxy formation set by the DM nature or by gas physics (or by both)?

#### **DM** nature and structure formation



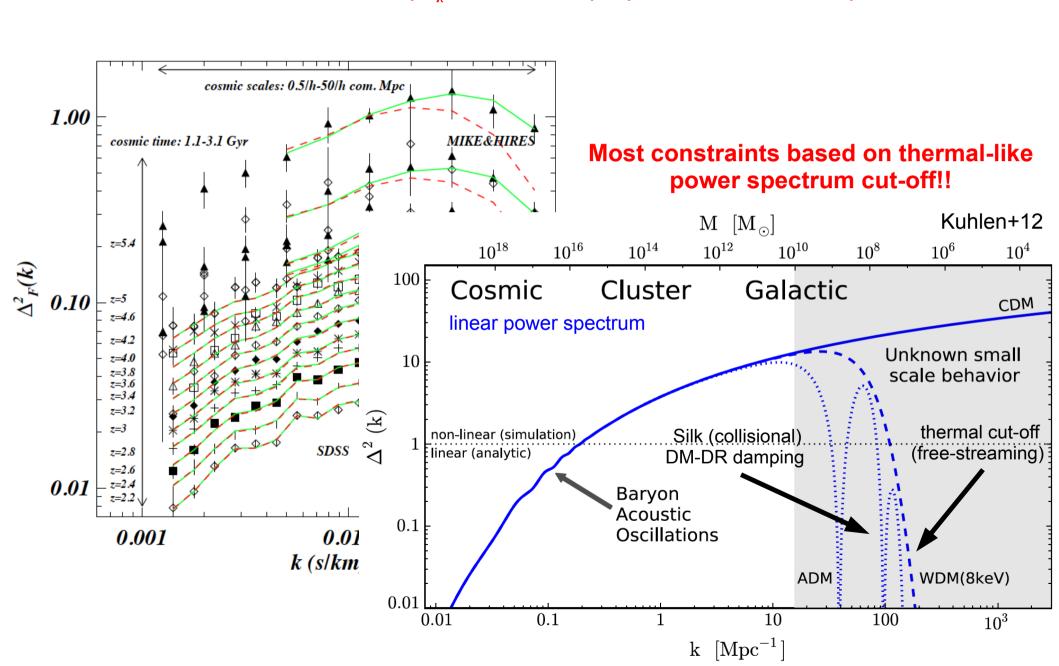
#### (e.g. Ly- $\alpha$ forest constraints)

Viel+13 thermal WDM ( $m_{\chi} > 3.3 \text{ keV}$ ,  $M_f(k_{1/2}) \sim 3x10^8 M_{Sun}$ ,  $2\sigma$  C.L.)



#### (e.g. Ly- $\alpha$ forest constraints)

Viel+13 thermal WDM ( $m_x > 3.3 \text{ keV}$ ,  $M_f(k_{1/2}) \sim 3x10^8 M_{Sun}$ ,  $2\sigma$  C.L.)



Onset of structure formation

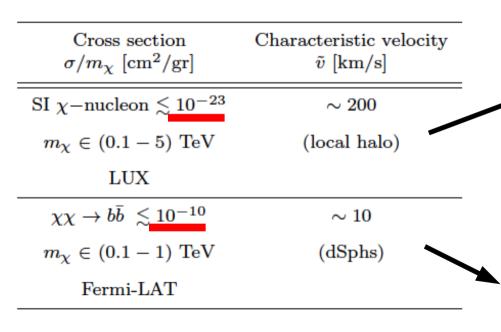
halo mass seed DM nature

DM interactions ?

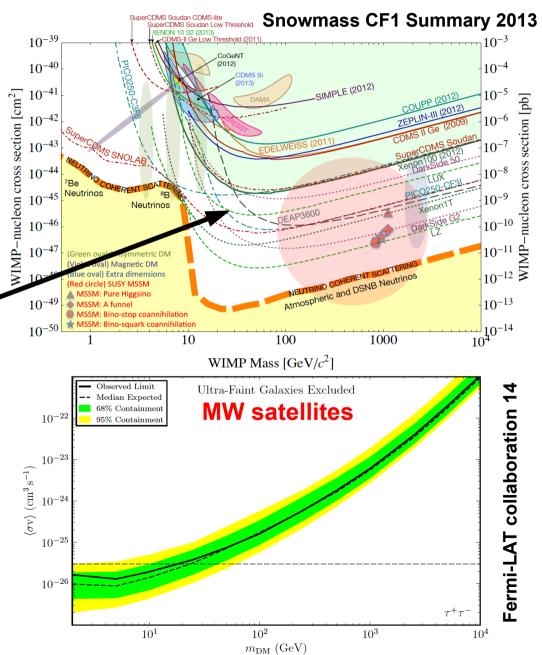
Are non-gravitational DM interactions irrelevant for galaxy formation?

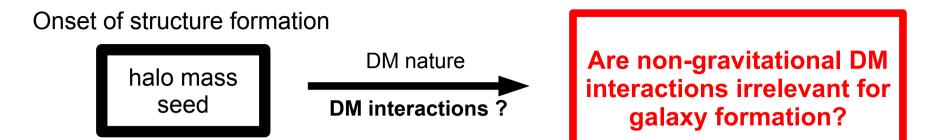
Are non-gravitational DM interactions irrelevant for galaxy formation?

DM particle interactions (weak scale) **hoped** by most detection efforts!!



1 cm<sup>2</sup>/g ~ 2 barns/GeV





DM particle interactions (weak scale) hoped by most detection efforts!!

Cross section $\sigma/m_{\chi}  [{\rm cm}^2/{\rm gr}]$	Characteristic velocity $\tilde{v}$ [km/s]
SI $\chi$ -nucleon $\lesssim 10^{-23}$	$\sim 200$
$m_{\chi} \in (0.1-5) \text{ TeV}$	(local halo)
LUX	
$\chi\chi o bar b \ \lesssim 10^{-10}$	$\sim 10$
$m_\chi \in (0.1-1)~{\rm TeV}$	(dSphs)
${\bf Fermi\text{-}LAT}$	

Does it interact with ordinary matter?

χ-nucleus interactions extremely low to impact structure information

Does it interact with itself (annihilation)?

χ-χ self-annihilation extremely low to impact structure information

1 cm<sup>2</sup>/g ~ 2 barns/GeV

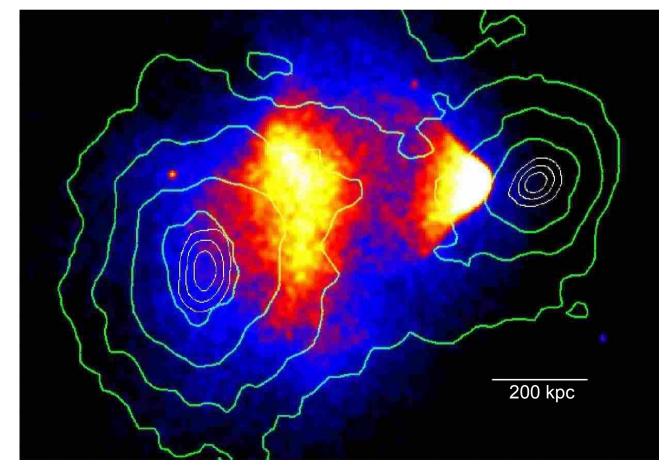
Onset of structure formation

halo mass seed DM nature

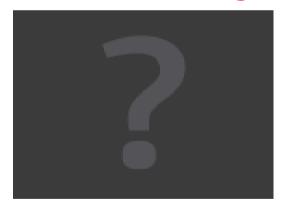
DM interactions ?

Are non-gravitational DM interactions irrelevant for galaxy formation?

Does it interact with itself (collisions)?



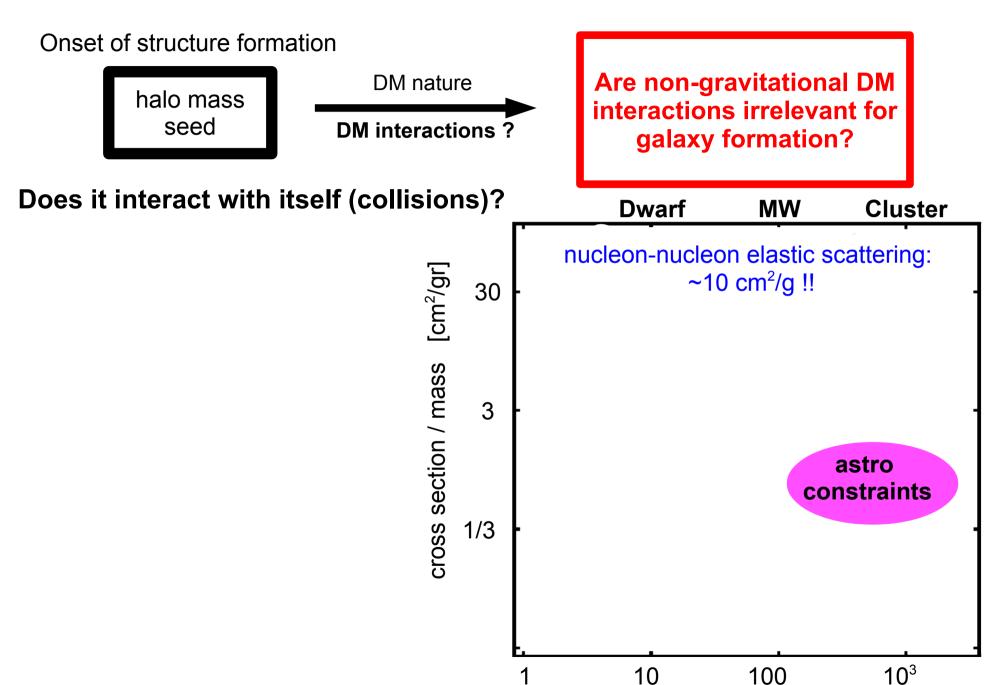
(Randall+08)  $\sigma/m < 1.25 \text{ cm}^2/\text{gr}$ 



**Credit: John Wise / KIPAC** 

Caveat: DM-only simulation gas and stars might weaken the constraint

Bullet Cluster (Clowe +06)



velocity dispersion [km/s]

Onset of structure formation

halo mass seed DM nature

DM interactions ?

Are non-gravitational DM interactions irrelevant for galaxy formation?

Does it interact with itself (collisions)?

Constraints allow collisional DM that is astrophysically significant in the center of galaxies:

Ć

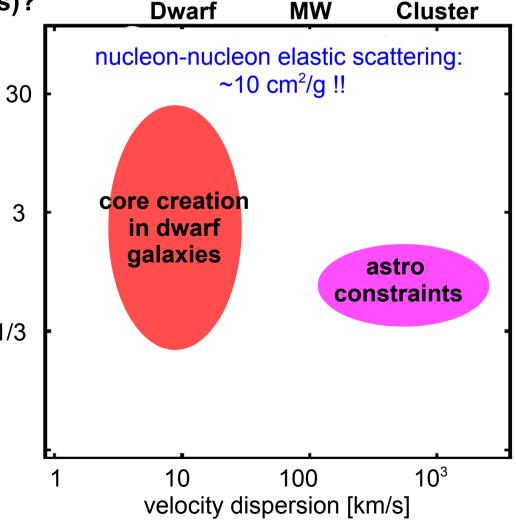
cross section / mass [cm²/gr]

Average scattering rate per particle:

$$\frac{\overline{R}_{sc}}{\Delta t} = \left(\frac{\sigma_{sc}}{m_{\chi}}\right) \overline{\rho}_{dm} \ \overline{v}_{typ}$$

~ <1 scatter/particle/t<sub>H</sub>>

Far from the fluid and collisionless regimes (Knudsen number =  $\lambda_{mean}/L > \sim 1$ )



Onset of structure formation

halo mass seed

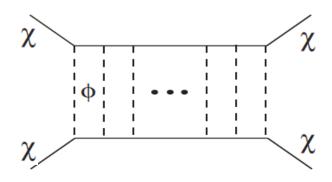
DM nature

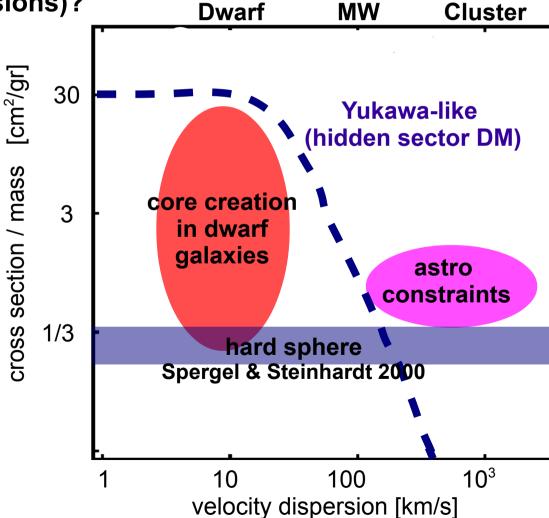
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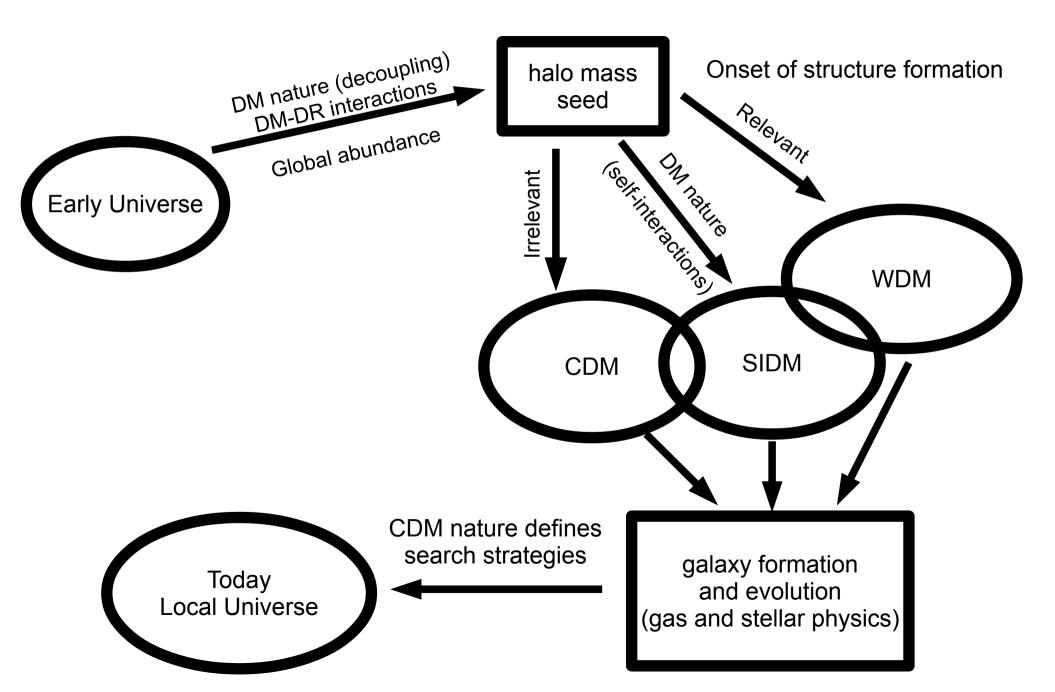
Does it interact with itself (collisions)?

velocity-dependence motivated by a new force in the "dark sector" (analogous to Rutherford scattering) e.g. Yukawa-like, Feng+09

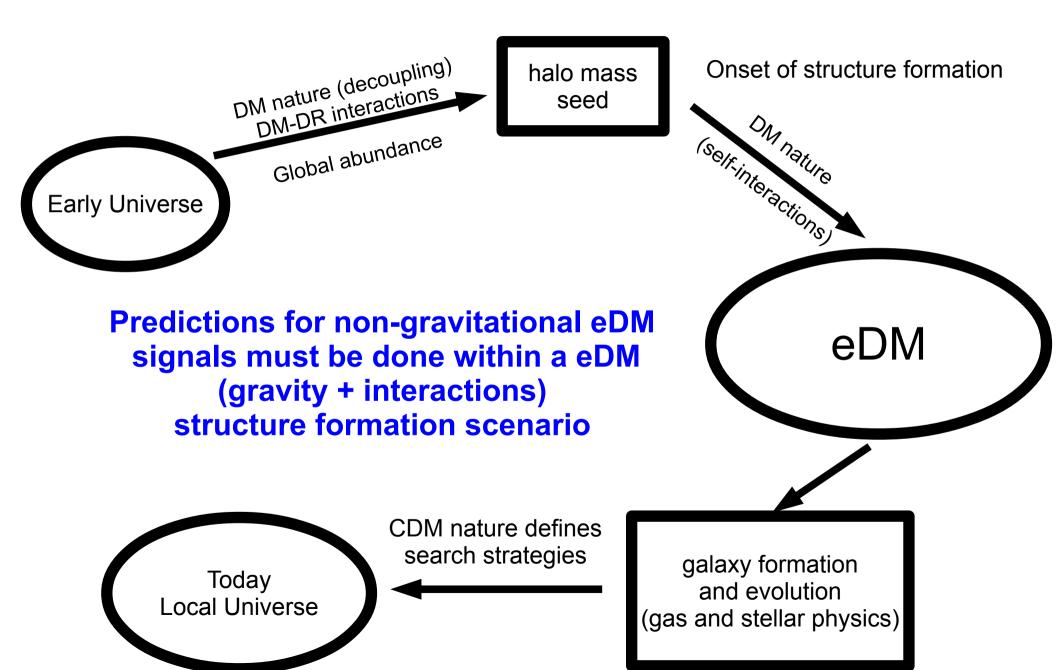




## Towards an effective theory of structure formation



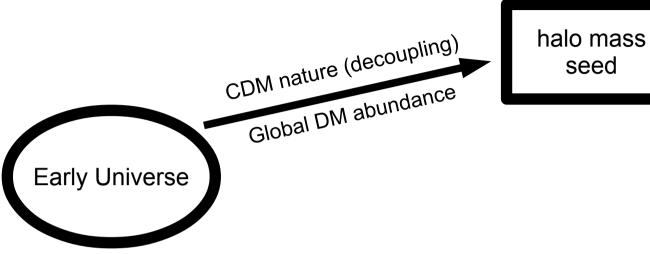
## Towards an effective theory of structure formation



# Using N-body simulations to obtain predictions for non-gravitational DM signals:

the CDM case

#### The relevance of the CDM nature across time



A guiding fundamental principle?

e.g. a new symmetry, SUSY

DM

SM

SM

SM

SM

SM

SM

SM

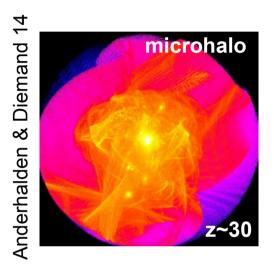
SM

**Multiple mechanisms** 

of CDM production

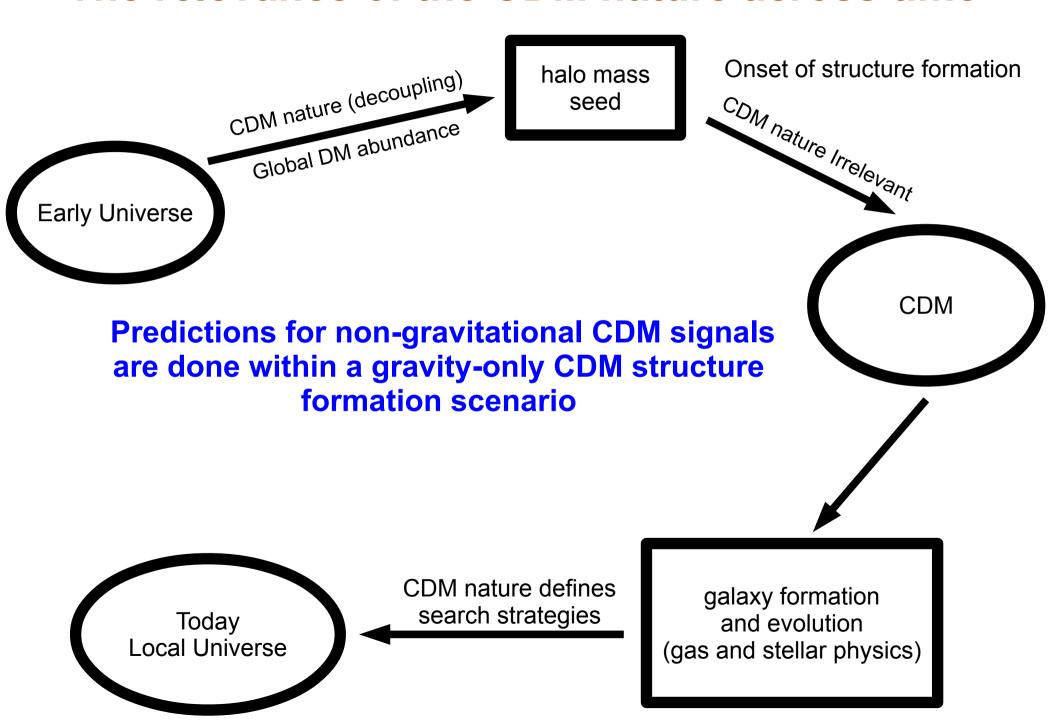
Weak-scale (100 GeV) thermal dark particles (WIMPs) "naturally" give the right DM abundance

Onset of structure formation



Minimum clustering scale (~10-6M₀ for "vanilla" WIMPs)

#### The relevance of the CDM nature across time



# DM signals from CDM N-body simulations

Any DM signal could be predicted knowing the local DM distribution in phase space at all points and at all times:

$$f(ec{x},ec{v},t)\mathrm{d}^3ec{x}\mathrm{d}^3ec{v}$$

In practice however, we can only compute, measure, the DM distribution averaged over a certain macroscopic scale (coarse-grained distribution)

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In practice however, we can only compute, measure, the DM distribution averaged over a certain macroscopic scale (coarse-grained distribution)

In N-body simulations the coarse-grained distribution is given by a discrete representation of N particles:

$$\hat{f}(\mathbf{x}, \mathbf{v}, t) = \sum_{i} (M_i/m) W(|\mathbf{x} - \mathbf{x}_i|; h_i) \delta^3(\mathbf{v} - \mathbf{v}_i)$$

macro-to-micro-particle mass ratio

each particle is smoothed in space to give a smooth local density each macro-particle travels at one speed

(needed to compute potential)

The system of N particles is advanced in time in a cosmological context (Andrea's talk)

$$n(x) = \int d^3 \mathbf{v} \hat{f}$$

### DM signals: standard approach

At a given time, split the phase space distribution in density and velocity distributions:

$$\hat{f}(\mathbf{x}, \mathbf{v}) \propto \hat{\rho}(\mathbf{x}) \hat{f}_v(\mathbf{v})$$

**Example 1: Indirect detection (DM self-annihilation)** 

Annihilation rate (# of events per unit time in a region of volume V)

$$R_{\rm ann} = \frac{1}{2m_\chi^2} \int_V d^3 \mathbf{x} \rho^2(\mathbf{x}) \langle \sigma_{\rm ann} v \rangle \qquad \text{"thermal" average} \\ \sim \int_{-\infty}^{\infty} (\sigma v_{\rm rel}) f(v_{\rm rel}) d^3 v_{\rm rel} \\ \qquad \qquad \qquad \text{particle physics}$$

#### Common method:

- Assume Maxwell-Boltzmann speed distribution
- Local density estimated from simulation

### DM signals: standard approach

At a given time, split the phase space distribution in density and velocity distributions:

$$\hat{f}(\mathbf{x}, \mathbf{v}) \propto \hat{\rho}(\mathbf{x}) \hat{f}_v(\mathbf{v})$$

#### Example 2: Direct detection (DM-nuclei scattering)

Scattering rate (# of events per unit time per unit mass of detector):

$$\frac{dN}{dE_r} = \frac{\sigma_0 \rho_\chi}{2\mu^2 m_\chi} F^2(q) \int\limits_{v_{\rm min}}^{v_{\rm esc}} \frac{f(v)}{v} dv$$
DM-nucleus reduced mass threshold energy of the detector

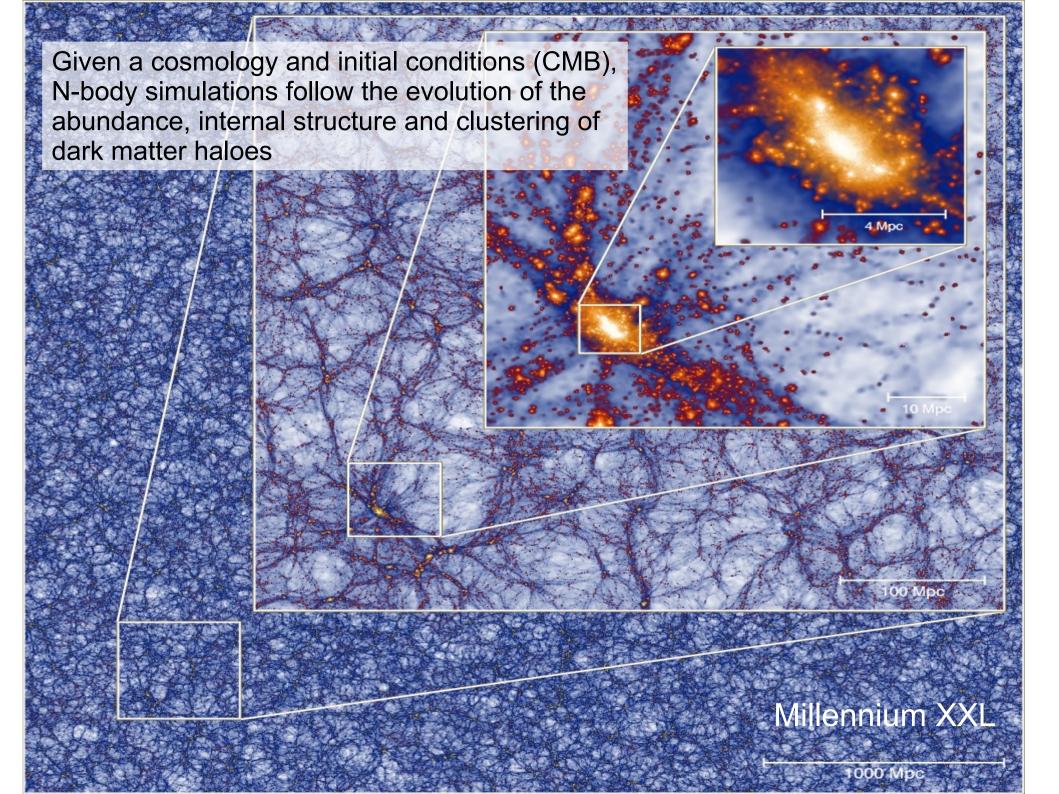
#### Common method:

- Assume Maxwell-Boltzmann speed distribution
- Local density estimate from observations

### N-body simulations and DM probes

		LS	SS	Halos		Substructure					Local					
from Kuhlen+12		voids, walls, filaments	halo mass functions	concentration-mass relation	halo shapes	density profiles	pseudo-phase-space density	mass (or V <sub>max</sub> ) functions	density profiles	central density	spatial distribution	streams	folds & caustics	local density	tidal streams	dark disk
Astrophysical	Dwarf galaxy abundance															
	Dwarf galaxy kinematics															
	Stellar streams															
	Gravitational lensing															
on	Extra-galactic DGRB															
	Galactic DGRB															
Indirect Detection	Clusters															
i se	Galactic Center															
Ğ	Milky Way Dwarfs															
ç	Dark Subhalos															
iji.	Local anti-matter															
	Neutrinos from Earth & Sun															
	Substructure boost								T							
	Sommerfeld boost															
Direct	"Vanilla" ~ 100 GeV DM															
	light / inelastic DM															
	axions															
	directionally sensitive experiments															

# What we know from N-body simulations: the resolved regime



#### A sample of state-of-the-art simulations

### Precise prediction of CDM distribution from ~Gpc to ~100 pc scales

Cosmic

Aquarius A-1

**GHalo** 

Via Lactea II

GADGET-3

PKDGRAV2

PKDGRAV2

#### DM-only simulations

Name	Name Code		$N_{p}$	$m_p$	$\epsilon_{ m soft}$					
		[h <sup>-1</sup> Mpc]	$[10^9]$	$[h^{-1} \; M_{\odot}]$	[h <sup>-1</sup> kpc]					
DEUS FUR	Ramses-Deus	21000	550	$1.2 \times 10^{12}$	$40.0^{\dagger}$					
Horizon Run 3	<b>G</b> отрм	10815	370	$2.5\times10^{11}$	150.0					
Millennium-XXL	Gadget-3	3000	300	$6.2 \times 10^9$	10.0					
Horizon-4∏	RAMSES	2000	69	$7.8 \times 10^{9}$	$7.6^{\dagger}$					
Millennium-II	Gadget-3	100	10	$6.9 \times 10^{6}$	1.0					
MultiDark Run1	Art	1000	8.6	$8.7 \times 10^{9}$	7.6 <sup>†</sup>					
Bolshoi	Art	250	8.6	$1.4 \times 10^{8}$	$1.0^{\dagger}$					
<sup>†</sup> For AMR simulations (Ramses, Art) $\epsilon_{\text{soft}}$ refers to the highest resolution cell width.										
Cluster										
Name	Code	L <sub>hires</sub>	$N_{p,hires}$	$m_{p,hires}$	$\epsilon_{ m soft}$					
		[h <sup>-1</sup> Mpc]	$[10^9]$	$[h^{-1} \; M_{\odot}]$	[h <sup>-1</sup> kpc]					
Phoenix A-1	Gadget-3	41.2	4.1	$6.4 \times 10^5$	0.15					
GALACTIC										
Name	C 1	т	N	122						
	Code	$L_{hires}$	$N_{p,hires}$	$m_{p,hires}$	$\epsilon_{ m soft}$					

5.9

3.89

4.86

 $4.3 \times 10^{9}$ 

 $2.1 \times 10^{9}$ 

 $1.0 \times 10^{9}$ 

 $1.7 \times 10^{3}$ 

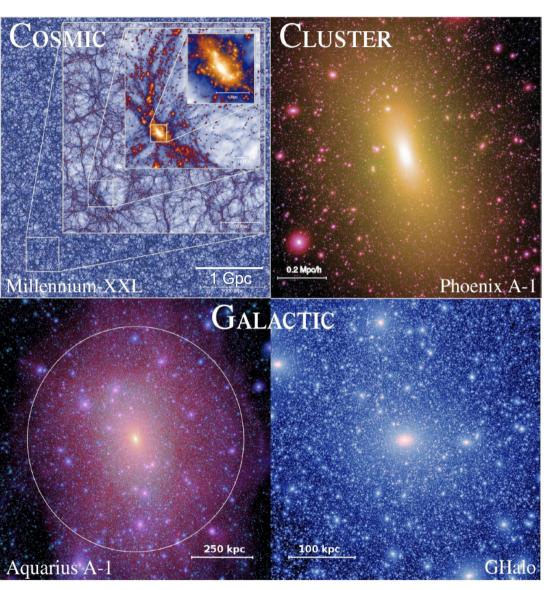
 $1.0 \times 10^{3}$ 

 $4.1 \times 10^{3}$ 

20.5

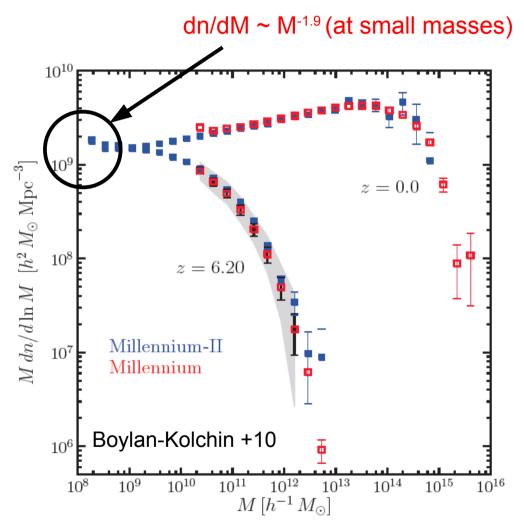
61.0

40.0



#### **Abundance of CDM haloes**

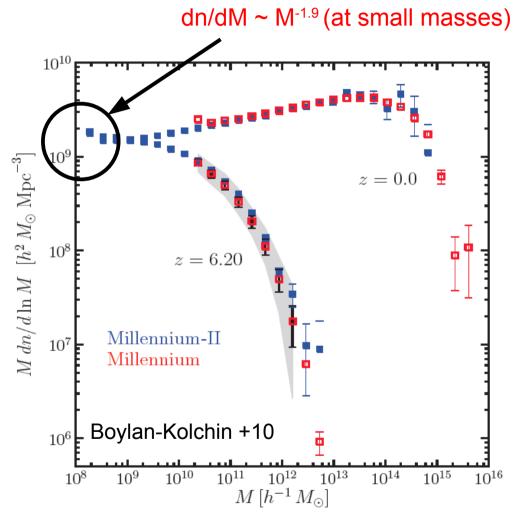
Mass function (dn/dM): number of haloes per comoving volume and per mass range. It evolves with redshift according to the CDM hierarchical scenario.



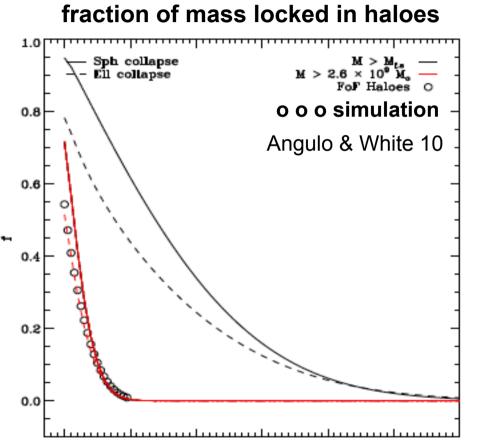
Minimum halo mass in CDM many orders of magnitude below mass resolution of current simulations!

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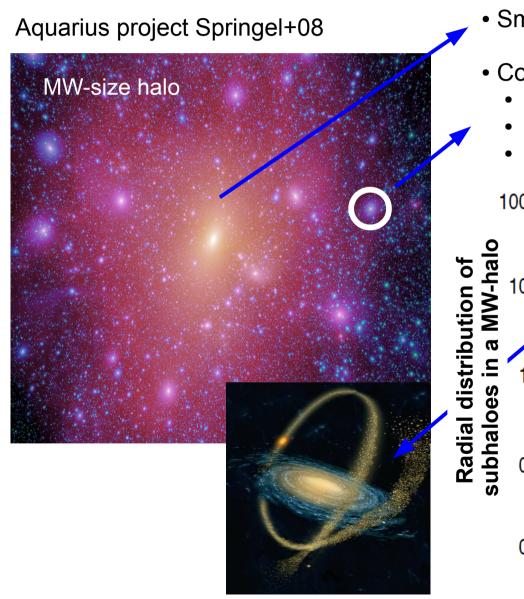
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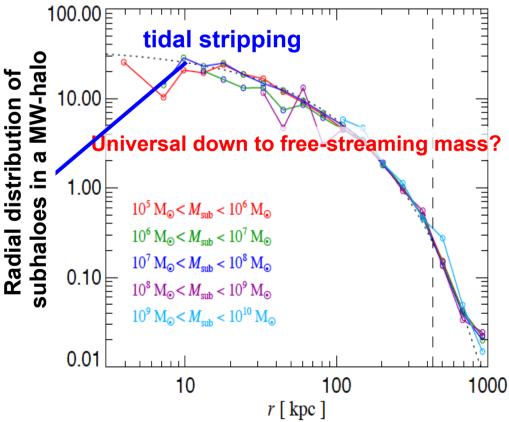
Not all dark matter is in haloes! but SM messengers from DM interactions should be produced more abundantly in haloes

#### Inner structure of CDM haloes

#### smooth distribution + substructures

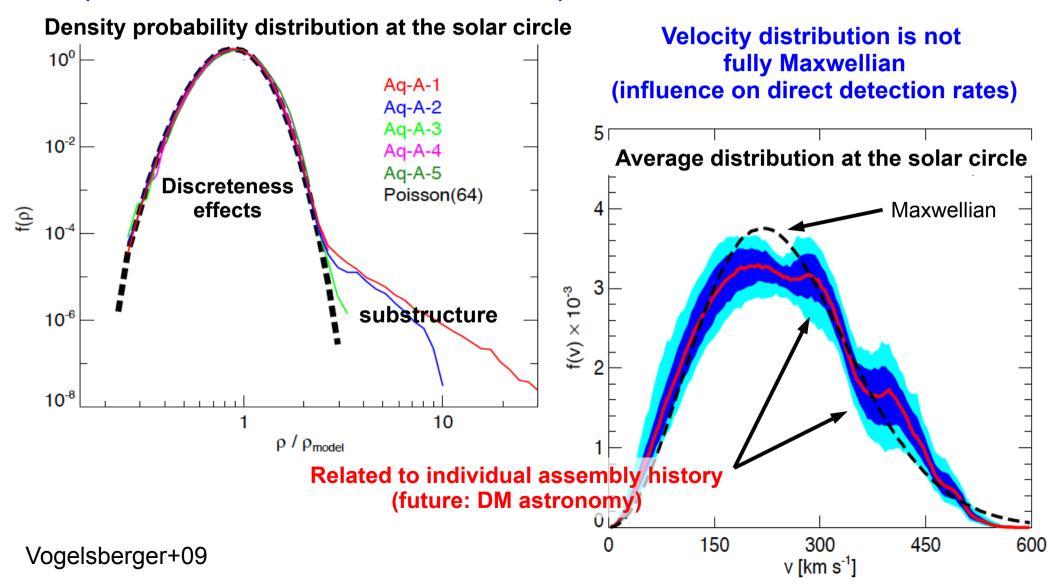


- Smooth spherical dist. (NFW or Einasto profile)
- Collection of subhaloes with a given:
  - Abundance (mass function)
  - Density profile (NFW or Einasto)
  - Radial distribution ("cored" Einasto)



#### CDM distribution at the solar circle

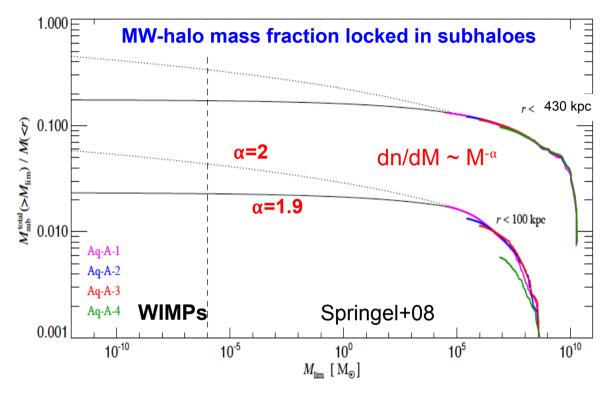
local (average) DM density distribution very smooth (chance of the Sun within a subhalo ~ 10-4)



#### Going beyond numerical resolution:

Uncertainties for predictions of DM signals in a CDM background

# Abundance and inner structure of unresolved haloes

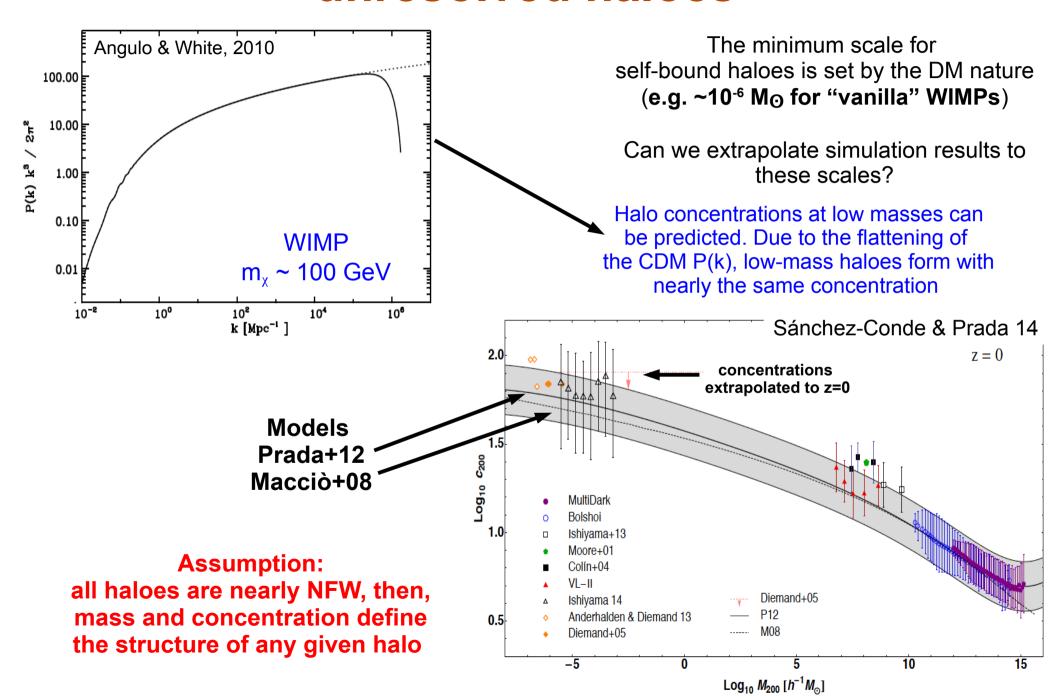


The minimum scale for self-bound haloes is set by the DM nature (e.g. ~10-6 Mo for "vanilla" WIMPs)

Can we extrapolate simulation results to these scales?

The abundance of unresolved (sub)haloes is one of the main uncertainties in many predicted CDM signals

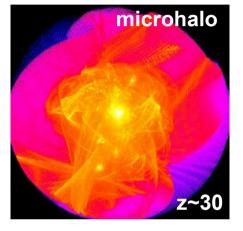
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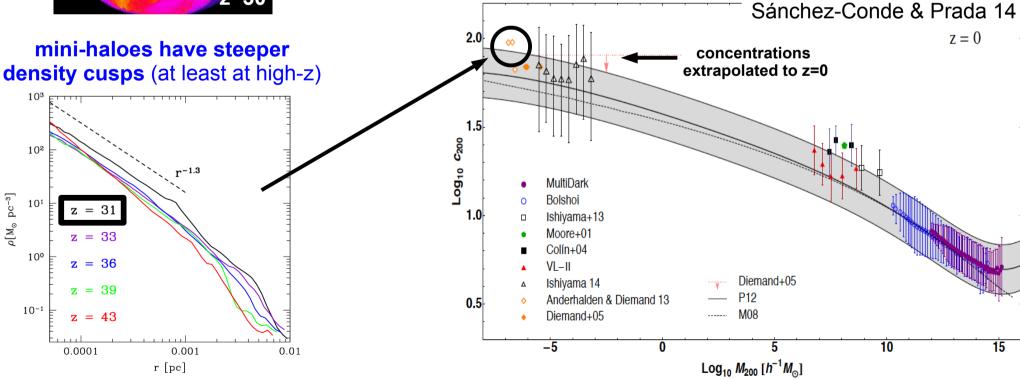


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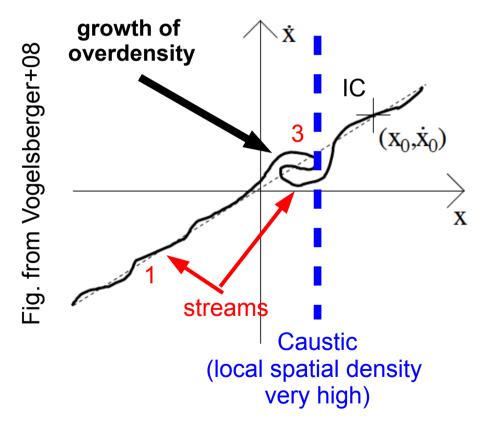




# The fine-grained phase-space distribution of DM haloes

Since CDM is cold and and collisionless, it lies in a 3D hypersurface in the 6D phase space (most of which is empty)

1D fine-grained distribution

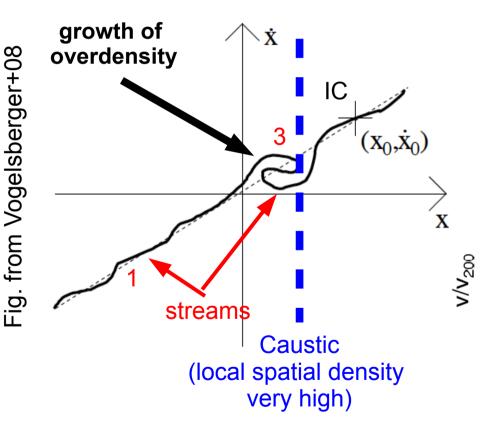


Thickness of line: primordial "thermal" velocity dispersion (width related to DM "coldness")

# The fine-grained phase-space distribution of DM haloes

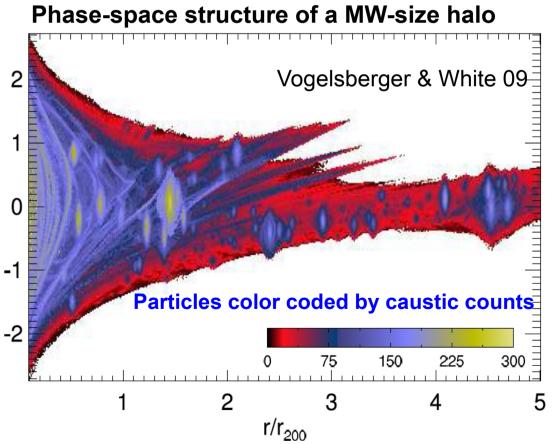
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1D fine-grained distribution



Thickness of line: primordial "thermal" velocity dispersion (width related to DM "coldness")

Method developed to identify streams and caustics in N-Body simulations



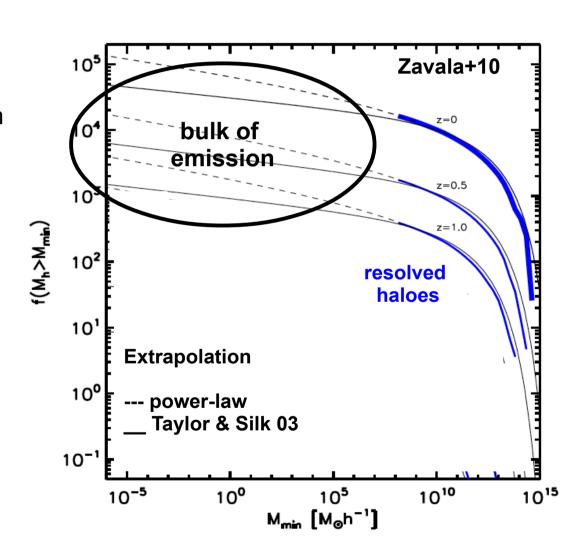
Since annihilation rates scale as  $\rho^2$ , any unresolved DM clumpiness should boost the annihilation rate

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Cosmic halo boost: excess emission from haloes over homogeneous background

$$f(M>M_{\min})=$$
 halo mass function 
$$\frac{1}{\bar{
ho}_B^2} \int_{M_{\min}}^{\infty} \left( \frac{\mathrm{d}n(M)}{\mathrm{dlog}M} \right) \bar{
ho}_h^2 V(M) f_h(V(M)) \mathrm{dlog}M$$
 emission per halo

Cosmic signals dominated by unresolved haloes!!



Since annihilation rates scale as  $\rho^2$ , any unresolved DM clumpiness should boost the annihilation rate

mass function **luminosity**  $B(M) = \frac{1}{L(M)} \int_{M_{min}}^{M} (dN/dm) [1 + B(m)] L(m) dm$ global subhalo boost: excess emission from subhaloes over that of a single halo sub-subhaloes (subdominant) halo luminosity Sky-map of the annihilation signal from the Via Lactea II -  $L(>M_{min}) \sim M^{-0.226}$  (Springel et al. 2008) MW-halo simulation  $10^{3}$ (Kuhlen, Madau & Silk 09)  $dn/dM \sim M^{-1.9}$  L  $\sim M^{0.75}$  (power law c(M))  $dn/dM \sim M^{-1.9}$  Bullock et al. (2001) c(M) Via Lactea II  $m L_{sub}(>M_{min})/L_{host}$ resolved subhaloes from Kuhlen+12 Boost up to the virial radius for a MW-halo  $10^{-10}$  $10^{-5}$  $10^{0}$  $10^{5}$  $10^{10}$  ${
m M}_{
m min}~[{
m M}_{\odot}]$ 

Since annihilation rates scale as  $\rho^2$ , any unresolved DM clumpiness should boost the annihilation rate

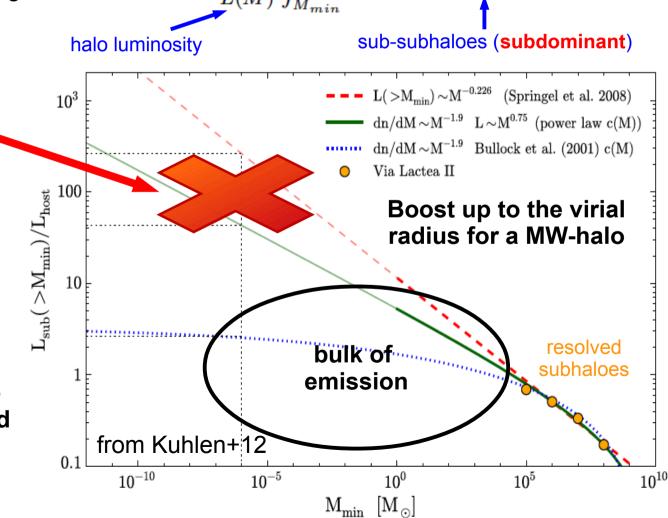
global subhalo boost: excess emission from subhaloes over that of a single halo

 $B(M) = \frac{1}{L(M)} \int_{M_{min}}^{M} (dN/dm) \; [1+B(m)] \; L(m) \; dm,$  uminosity sub-subhaloes (subdominant)

Models based on power-law extrapolations of c(M) are wrong!!

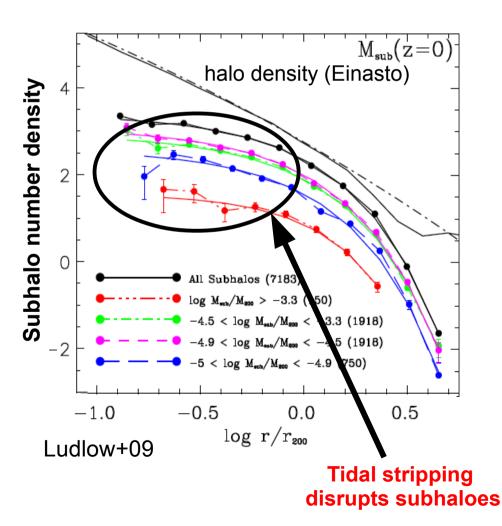
Important to consider the flattening of the power spectrum

Halo signals (integrated up to the virial radius) are dominated by unresolved subhaloes!!



# Since annihilation rates scale as $\rho^2$ , any unresolved DM clumpiness should boost the annihilation rate

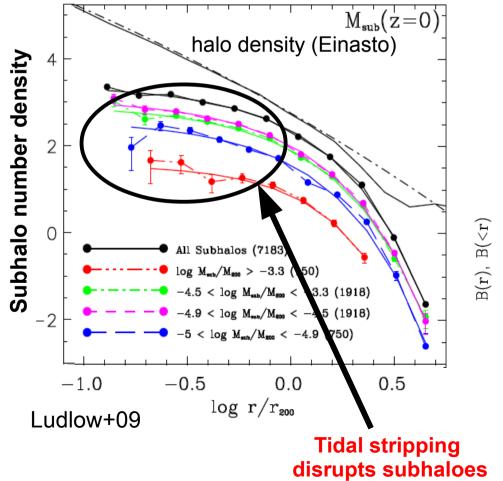
local (average) subhalo boost: local excess emission from subhaloes over that of the smooth halo emission

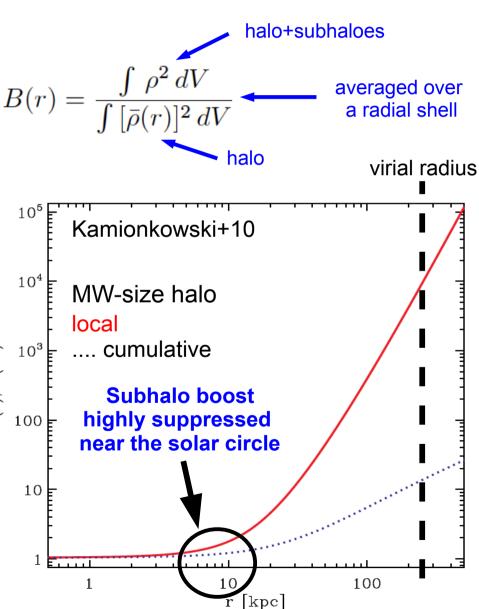


$$B(r) = \frac{\int \rho^2 \, dV}{\int [\bar{\rho}(r)]^2 \, dV} \text{ averaged over a radial shell}$$

# Since annihilation rates scale as $\rho^2$ , any unresolved DM clumpiness should boost the annihilation rate

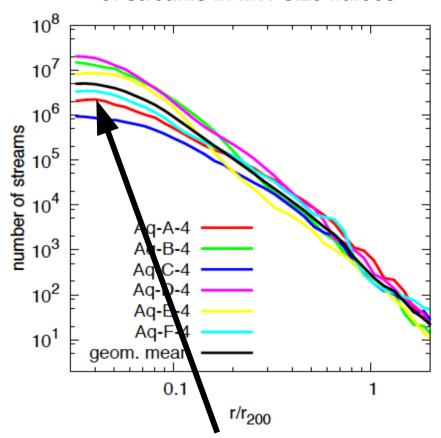
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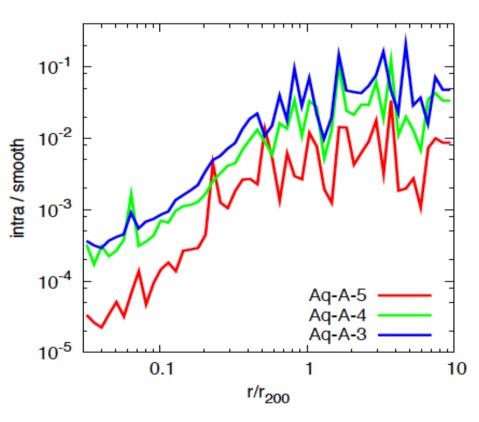
### The role of streams and caustics?

## Radial profile of the median number of streams in MW-size haloes



Very large number near the solar circle.
In most direct detection experiments,
it is "safe" to assume that the local velocity
distribution is smooth

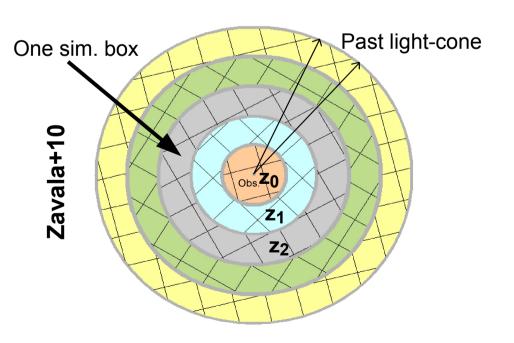
## Radial profile of the boost to the annihilation rate from caustics

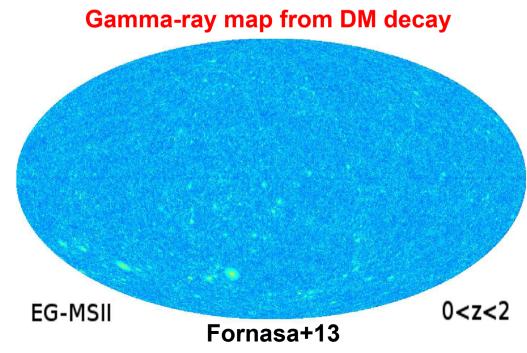


The annihilation boost by caustics is only important (but still subdominant) near the virial radius

# Example: recipe to predict cosmic DM annihilation/decay signals

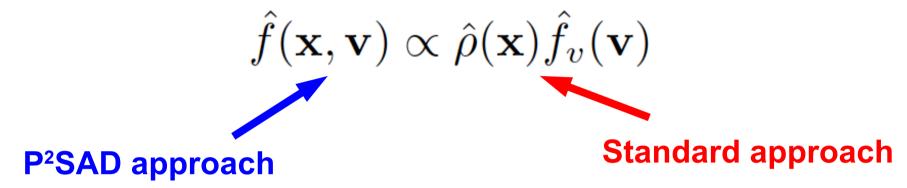
- 1) Take a large-scale simulation ~100Mpc (e.g. Millennium, Bolshoi)
- 2) Use resolved (sub)halo catalogues  ${}^{8}M_{0}$  to get abundances and global properties (mass, concentration,..)
- 3) Assume a density profile for each (sub)halo (calibrated from higher resolution simulations)
- 4) Extrapolate to unresolved masses (mass function, concentration-mass)
- 5) Produce a sky-map built on light-cones





# A new approach to predict non-gravitational DM signals: P<sup>2</sup>SAD

Use the full phase-space distribution information



**Advantages:** 

a novel approach on DM clustering

built-in phase-space information is used to extrapolated to the unresolved regime (through a physically-motivated model)

naturally accounts for DM signals that have velocity-dependent cross sections (e.g. Sommerfeld-enhancement)

## A new approach: P<sup>2</sup>SAD

**Example: Indirect detection (DM self-annihilation)** 

Annihilation rate (# of events per unit time in a region of volume V)

$$\begin{split} R_{\rm ann} &= \lim_{\Delta x \to 0} \left[ \frac{1}{2m_\chi^2} \int_V d^3\mathbf{x} \int d^3\mathbf{v} d^3\mathbf{\Delta} \mathbf{v} (\sigma v)_{\rm ann} f(\mathbf{x}, \mathbf{v}) f(\mathbf{x} + \mathbf{\Delta} \mathbf{x}, \mathbf{v} + \mathbf{\Delta} \mathbf{v}) \right] \\ &= \frac{1}{2m_\chi^2} \int d^3\mathbf{\Delta} \mathbf{v} (\sigma v)_{\rm ann} M_V \lim_{\Delta x \to 0} \Xi(\Delta x, \Delta v) \end{split}$$

Particle Phase Space Average Density  $(P^2SAD)$ 

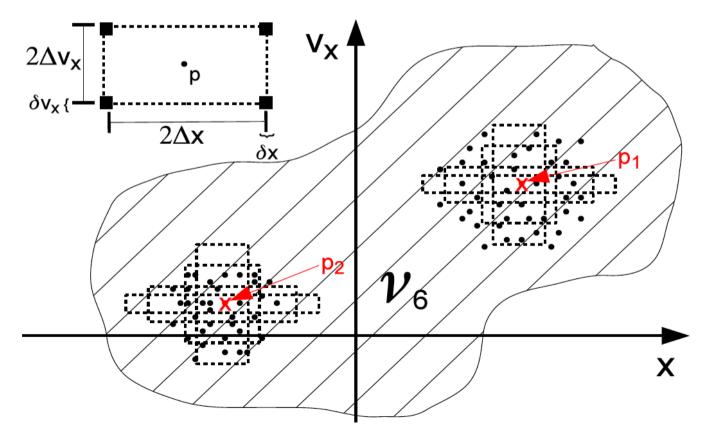
$$\Xi(\Delta x, \Delta v) \propto 2D$$
 phase – space 2PCF

## A new approach: P<sup>2</sup>SAD

Particle Phase Space Average Density  $(P^2SAD)$ 

 $\Xi(\Delta x, \Delta v) \propto 2D$  phase – space 2PCF

Estimator in an N-body simulation:  $\Xi(\Delta x, \Delta v)_{\text{sim}} = \frac{m_p \langle N_p(\Delta x, \Delta v) \rangle v_6}{V_6(\Delta x, \Delta v)}$ 



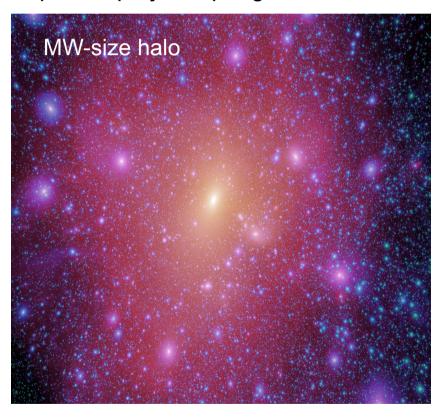


Zavala & Afshordi 14a, 14b

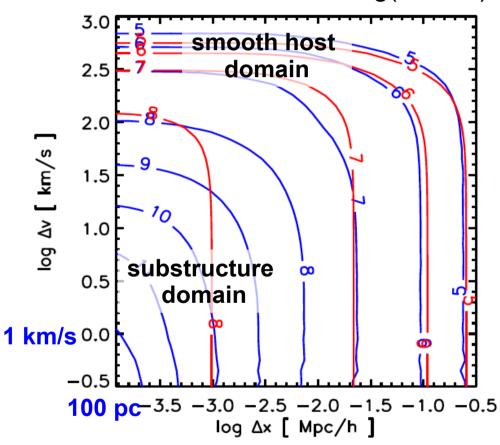
## A new approach: P<sup>2</sup>SAD

### A novel way to look at DM clustering

Aquarius project Springel+08



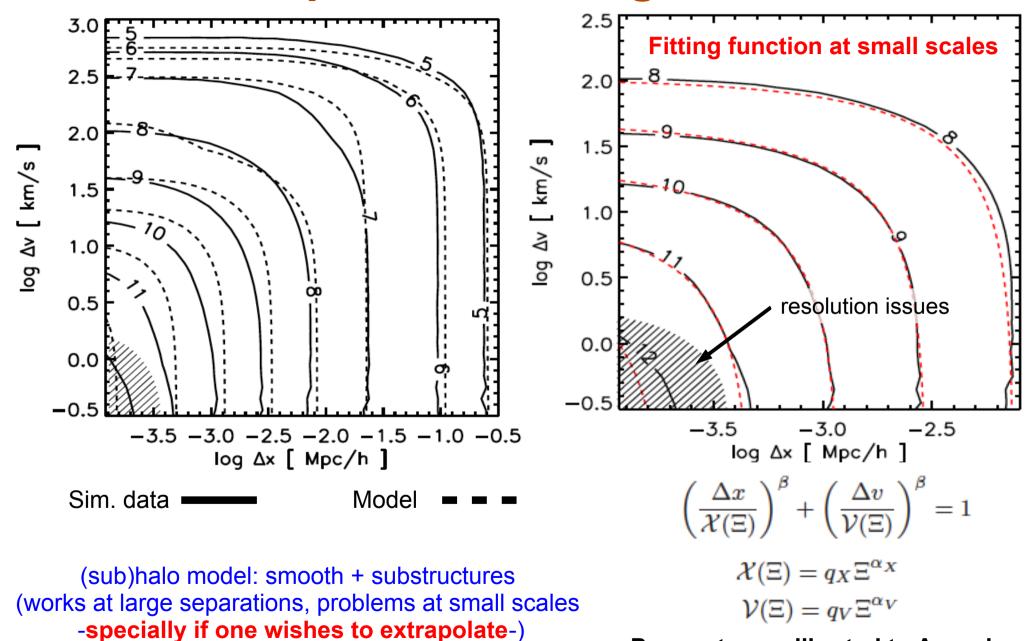
Contours of constant log(P<sup>2</sup>SAD)



red: Einasto-fit to smooth Aquarius halo

blue: full DM distribution

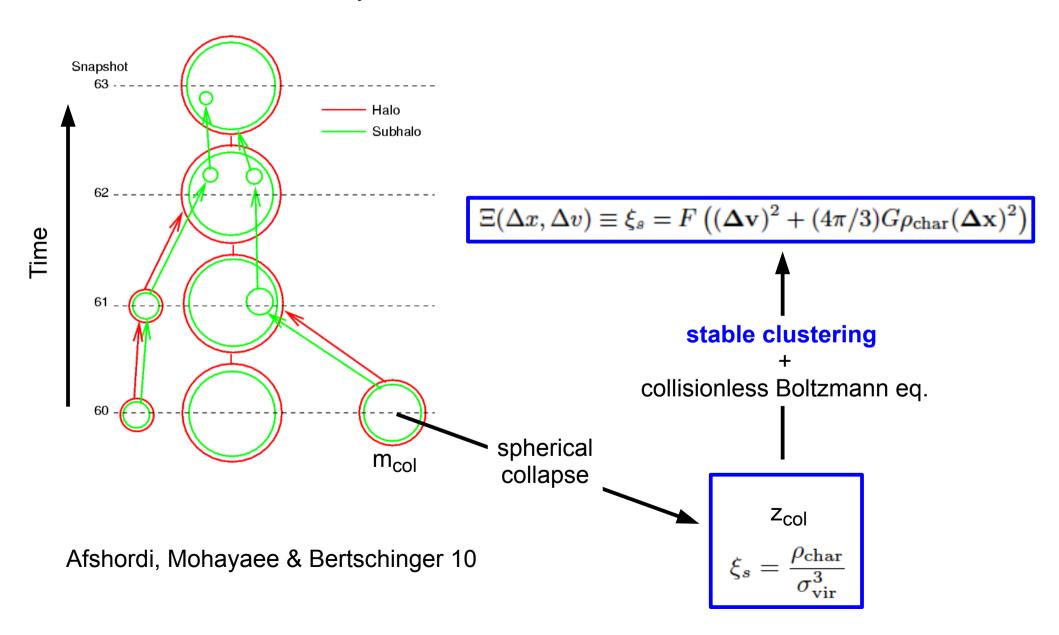
## Descriptive modelling of P<sup>2</sup>SAD

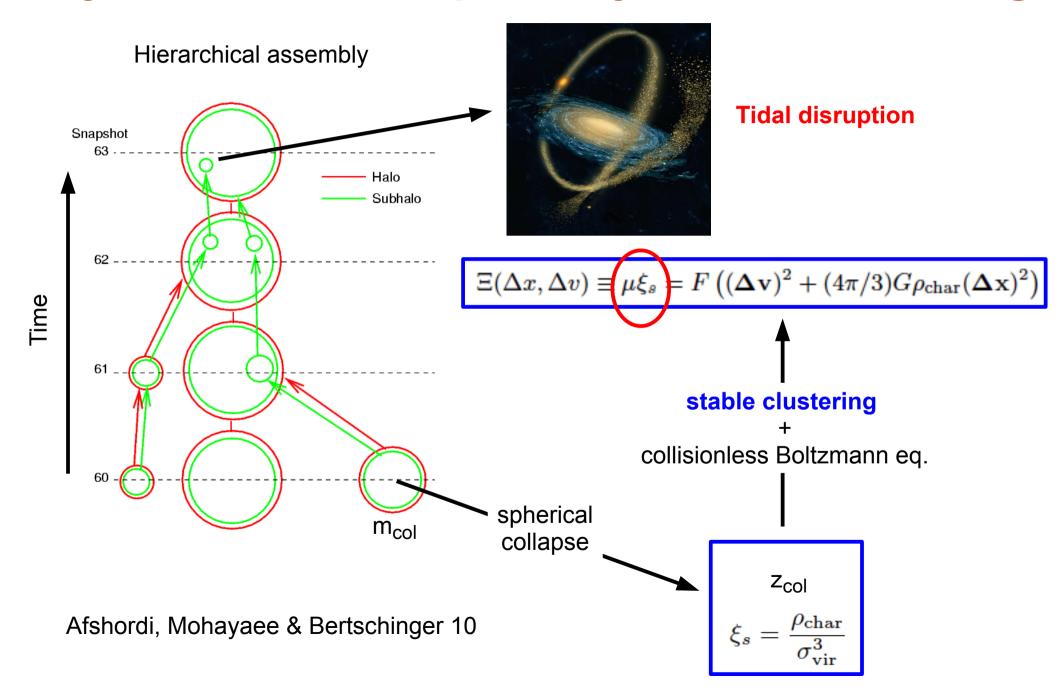


**Parameters calibrated to Aquarius** 

Hypothesis originally proposed by Davis & Peebles 1977. Extension to phase space: "the number of particles within the physical velocity  $\Delta v$  and physical distance  $\Delta x$  of a given particle does not change with time for small enough  $\Delta v$  and  $\Delta x$ "

Hierarchical assembly





#### **Deviations from stable clustering**

$$\left(\frac{\Delta x}{a\lambda(m_{\rm col})}\right)^{\beta} + \left(\frac{\Delta v}{b\zeta(m_{\rm col})}\right)^{\beta} = 1$$

λ and ζ are given by spherical collapse

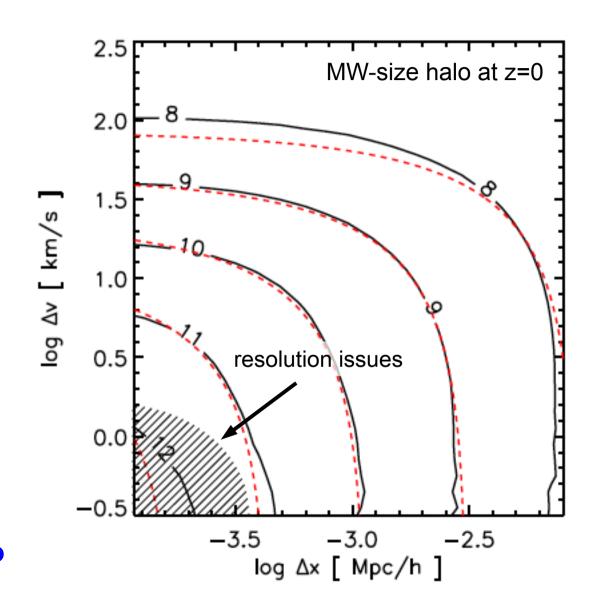
a, b and  $\beta$  slowly varying functions of redshift of order 1

We propose a tidal disruption model

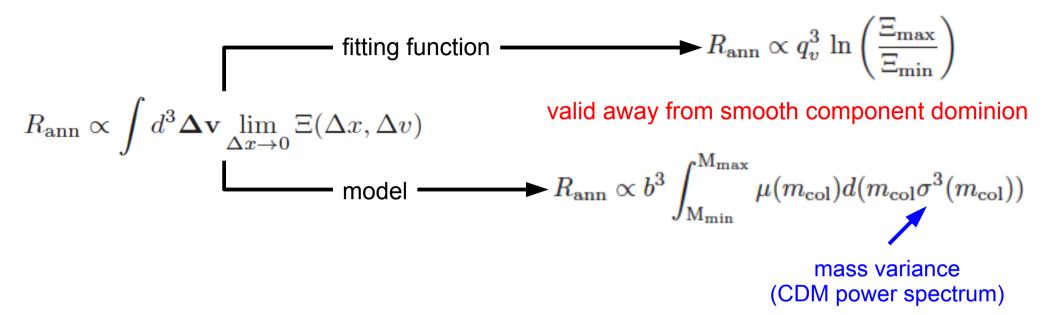
$$\mu(m_{\rm col}; z)\xi_s = \Xi(\Delta x, \Delta v)$$

#### **Parameters calibrated to Aquarius**

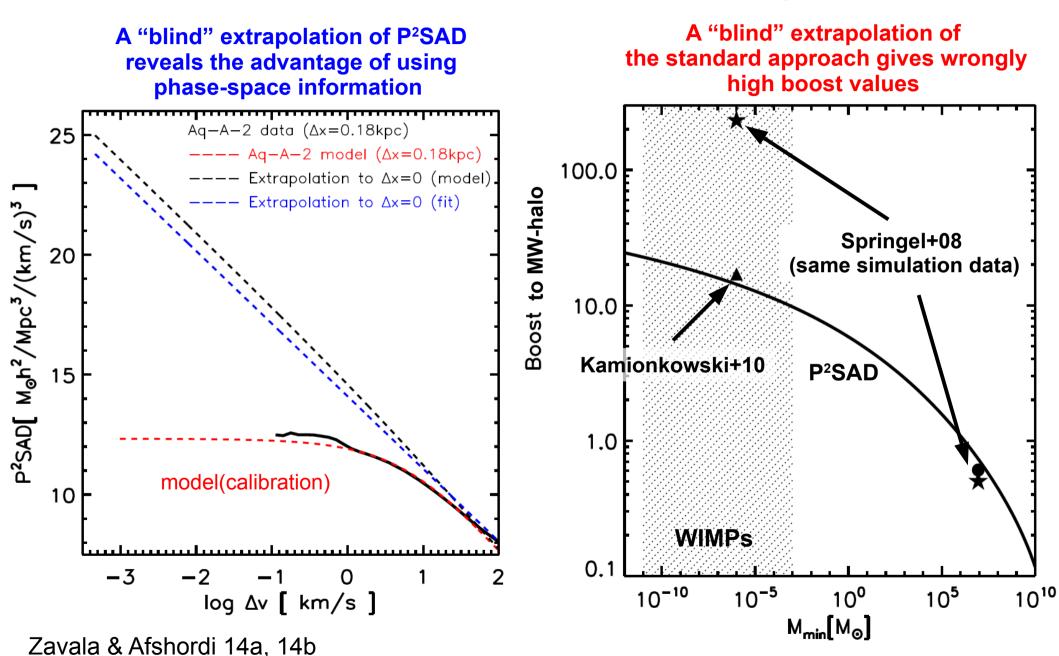
Physically-motivated model to compute DM signals down to unresolved scales!!



# (e.g. global substructure boost to annihilation: (σν)<sub>ann</sub> = cte)



# (e.g. global substructure boost to annihilation: (σν)<sub>ann</sub> = cte)



## **Concluding remarks**

- CDM is by itself an incomplete DM theory, it needs completion with a particle physics model (all beyond SM: "exotic")
- Decisive decade for "standard" DM model (CDM + WIMPs): experiments reaching the "expected" WIMP cross sections (LHC, Fermi, LUX,...)
- An effective (more generic) theory of structure formation must consider a broader range of allowed DM phenomenology (DM interactions, different P(k)...) coupled with our developing knowledge of galaxy formation/evolution

## **Concluding remarks**

- Current CDM simulations cover a vast dynamical range giving accurate predictions
  of the DM distribution (~100 pc to ~1Gpc, ~1km/s to ~1000 km/s). They are our
  most accurate method to predict non-gravitational DM signals in the resolved regime
- Still, many signals are sensitive to smaller scales, far from current resolutions. Extrapolations of several orders of magnitude are needed to predict these signals.
- Physically-motivated models calibrated to simulations must be used to extrapolate
   The challenge for future simulations lies in testing these models
- The synergy between DM and baryons make many expected signals highly uncertain!!

## Remarks on future sims and DM signals

DM SIGNALS	MAIN UNCERTAINTY	FUTURE SIMULATIONS
Cosmic backgrounds	halo mass function and inner structure down to M <sub>min</sub>	simulate microhaloes up to z=0 (in voids maybe?)
Extragalactic haloes (extended): e.g. clusters	subhalo mass function and radial distribution down to M <sub>min</sub>	follow microhaloes as they orbit the host in highly clustered regions!
Galactic subhaloes	luminous: synergy with baryons dark: their abundance and radial distribution	full hydro sims of satellite galaxies (subgrid physics) *maybe future obs. (proper motions of stars) can constrain the DM distribution*
Galactic Centre	Synergy with baryons	As above
Local (direct detection)	DM phase-space clustering near Earth	physically motivated models calibrated with parsec scale simulations

 $M_{min}$  = minimum self-bound mass (set by DM free streaming, kinetic decoupling)