Lecture 2

The Cold Dark Matter (CDM) paradigm

Outline:

- The post-recombination power spectrum
- The Cold Dark Matter (CDM) paradigm: main hypotheses

  i) DM is cold

  ii) DM is collision-less
- Successes of the CDM model (large-scales)
- Striking predictions of the CDM model:

  i) The abundance of DM halos

  ii) The universal density profile of DM halos

Literature: **(i)** Galaxy formation and evolution, Mo, van den Bosch & White, C. U. Press, 2010 **(ii)** Cosmological Physics, Peacock, C. U. Press, 1998 **(iii)** The large-scale structure of the Universe, Springel, Frenk & White, 2006, Nature, 440, 7088 **(iv)** Resolving cosmic structure formation with the Millennium-II Simulation, Boylan-Kolchin et al. 2010, MNRAS, 398, 1150 **(v)** The Aquarius Project: the subhaloes of galactic haloes, Springel et al. 2008, MNRAS, 391, 1685

The standard DM paradigm: statistical properties of the post-recombination Universe

- Since the primordial density field is believed to be a random field generated by some random processes, models should be compared with observations in a statistical sense.

- The time evolution of individual perturbations (represented by a single Fourier mode) is generalized to a probability distribution function (PDF) $P_x(\delta_1,\delta_2,.....\delta_n)d\delta_1 d\delta_2....d\delta_n$ that gives the probability that the field of density perturbations ($\delta=\rho/\langle\rho\rangle -1$) has values δ_i to $\delta_i+d\delta_i$ at positions x_i.

- Statistically, we are interested in the "moments" of the distribution. The 1st moment, the mean, is zero by definition, the 2nd moment, the variance $\sigma^2=\langle\delta^2\rangle$ is the quantity of interest, together with the two-point correlation function (2PCF) $\xi(x)=\langle\delta_1\delta_2\rangle$ with $x=|\mathbf{x}_1-\mathbf{x}_2|$ (notice that it only depends on the distance between the two points, and that $\xi(x=0)=\sigma^2$).

- We can equally represent the perturbation field in Fourier space. In this case the power spectrum P(k) is the Fourier transform of the 2PCF.

- If the density field is a Gaussian random field, then it is completely determined by the 2PCF, or by the power spectrum P(k).

- The initial power spectrum is commonly assumed to be a power law: $P(k) \alpha k^n$ (motivated for example by inflation where the initial perturbations result from quantum fluctuations to the inflaton field).

The standard DM paradigm: the post-recombination power spectrum

- The initial power spectrum is modified by a couple of physical effects prior to recombination:

- The Mészáros effect: when the energy density of the Universe is dominated by radiation, subhorizon perturbations stagnate:

$$\ddot{\Delta}_X + 2H\dot{\Delta}_X \approx 4\pi G\rho_r\Delta_X \qquad \boxed{\Delta_X \propto 1 + \frac{3}{2}\left(\frac{a}{a_{\rm eq}}\right)} \qquad \rho_r \gg \rho_X \gg \rho_B; \quad \rho_r \propto a^{-4}$$

- For superhorizon perturbations the Newtonian treatment is not valid. The horizon is given by the distance light can travel since the Big Bang to a given cosmic time; scales larger than the horizon are causally disconnected. Perturbations grow as a^2 during radiation dominion.

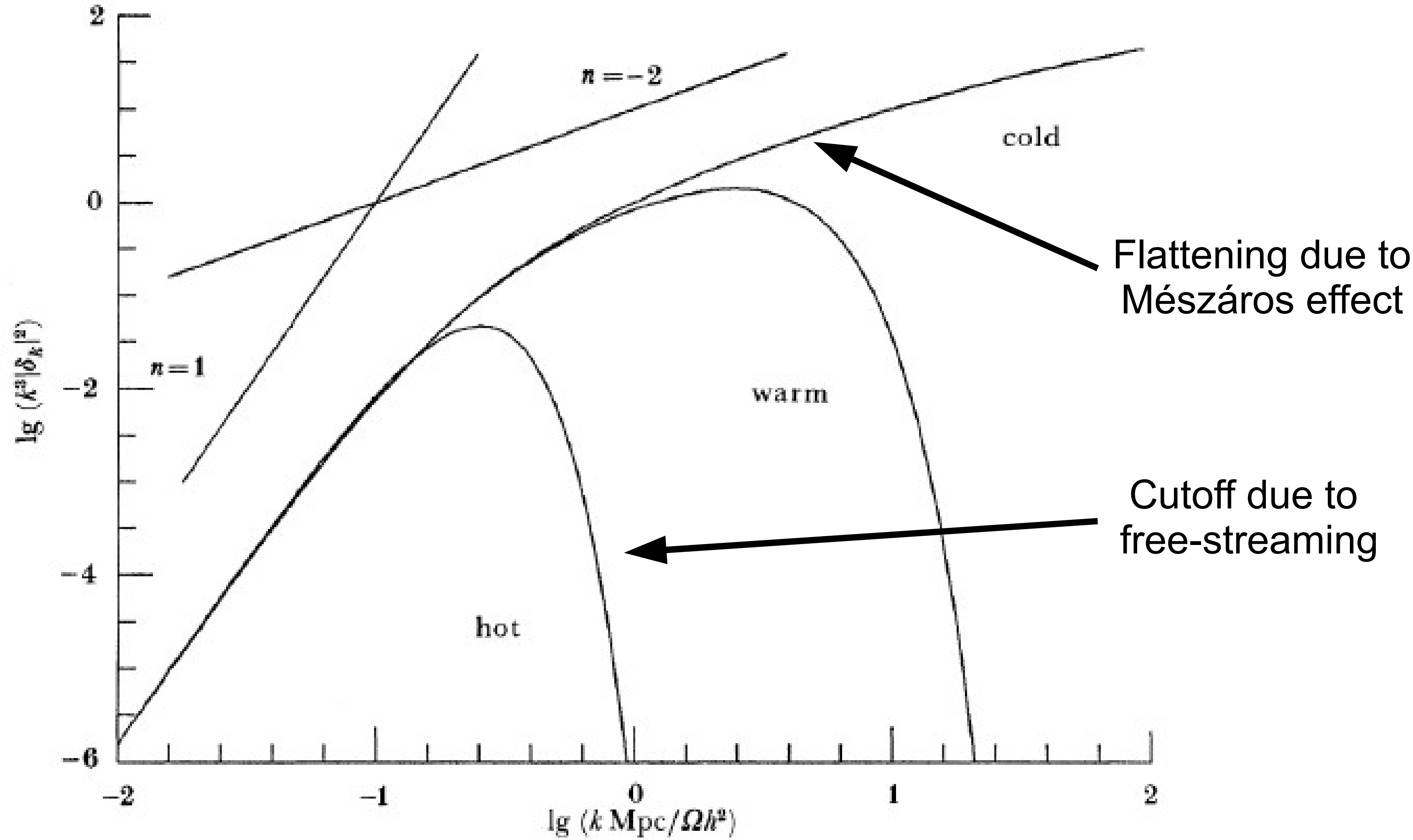
- Free-streaming (collisionless damping): damping of the density perturbation at small scales due to the random motions of DM particles (they move from crests to valley of the density field for perturbations that haven't become Jeans unstable):

$$R_{\rm fs} = \int_0^t \frac{v_{\rm pec}(t')}{a(t')}dt' = \frac{2c\,t_{\rm nr}}{a_{\rm nr}}\left[1 + \ln\left(\frac{a_{\rm eq}}{a_{\rm nr}}\right)\right], \quad ({\rm radiation-dominated:}\ \ a \propto t^{1/2},\ {\rm and}\ t_{\rm nr} < t_{\rm eq})$$

- Equality happens roughly at z_{eq}~2×10^4. Particles become non-relativistic roughly when $T=T_{nr}$ ~ m_X; $t_{nr} \alpha\ 1/m_X^2$, thus, the free-streaming scale is smaller for heavier particles.

The standard DM paradigm: the post-recombination power spectrum

Fig. from Frenk, C.S., 1986



DM models are generically classified according to their characteristic free-streaming length: Hot DM (m_X in the eV scale), Warm DM (m_X in the keV scale) and Cold DM (m_X in the GeV scale)

The standard DM paradigm: the post-recombination power spectrum

DM models are generically classified according to their characteristic free-streaming length: Hot DM (m_X in the eV scale), Warm DM (m_X in the keV scale) and Cold DM (m_X in the GeV scale)

- Hot DM is ruled out as the main component of DM since $M_{fs} \sim 10^{15} M_{Sun}$ (cluster-size) for $m_X \sim 30$eV. This contradicts the standard picture of structure formation.

- In Cold DM, structures form hierarchically: small perturbations collapse first. The variance of a perturbation of mass 'M' goes as:

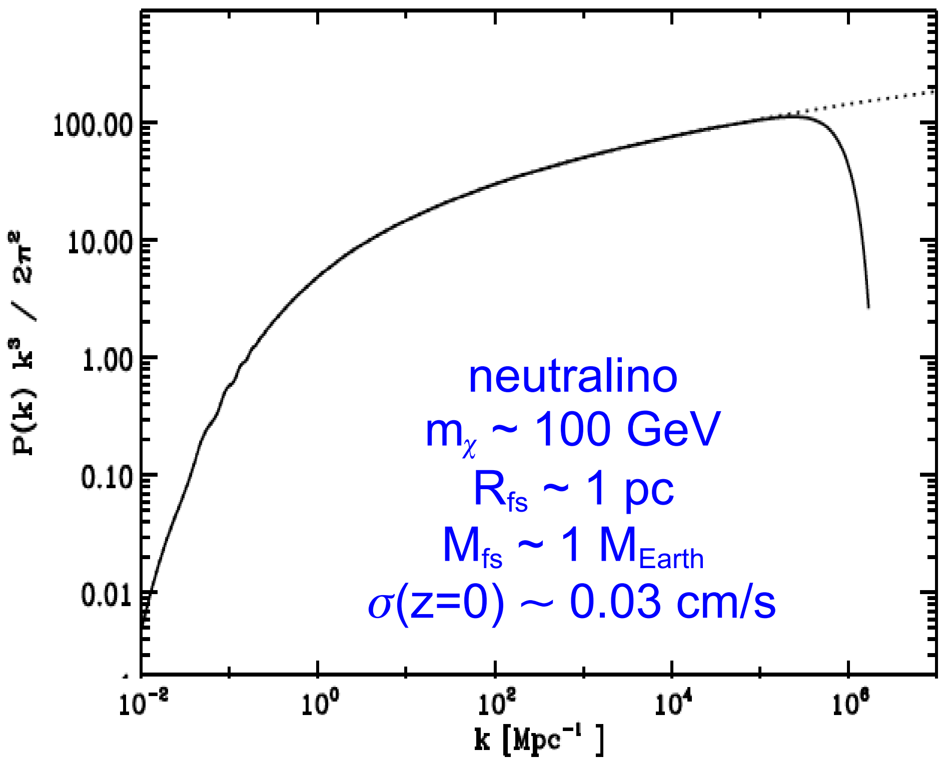
$$\sigma(M) \propto D(a) M^{-(3+n)/6}, \quad \text{with } D(a) \propto a(t) \ \ \text{(for EdS Universe)} \longrightarrow t_{\rm col} \propto M_{\rm col}^{(3+n)/4}$$

- The typical free-streaming length for CDM is ~ 1 Earth Mass ($m_X \sim 100$GeV, WIMPs)

- For Warm DM, structure formation is also hierarchical but $M_{fs} \sim 10^{10} M_{Sun}$ (dwarf-size); for $m_X \sim 1$keV.

The Cold Dark Matter (CDM) paradigm: DM is cold

Angulo & White, 2010



Free streaming length: $R_{\mathrm{fs}} \propto m_\chi^{-1}$

Velocity dispersion (unclustered matter):

$$\sigma_\chi \propto a^{-1} m_\chi^{-1/2}$$

How cold is DM?

Ultimately constrained by observations of the power spectrum at small scales

The best constraints come from the Ly-α forest (absorption features in the spectra of quasars)

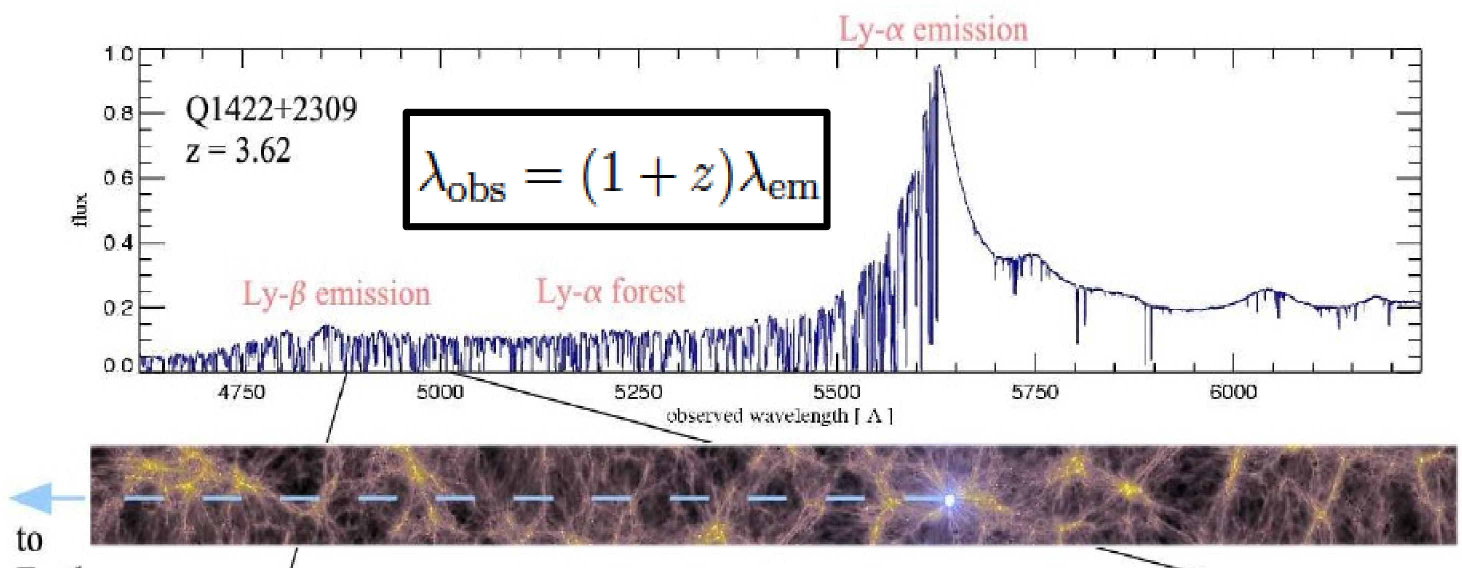


Fig. From Springel et al. 2006

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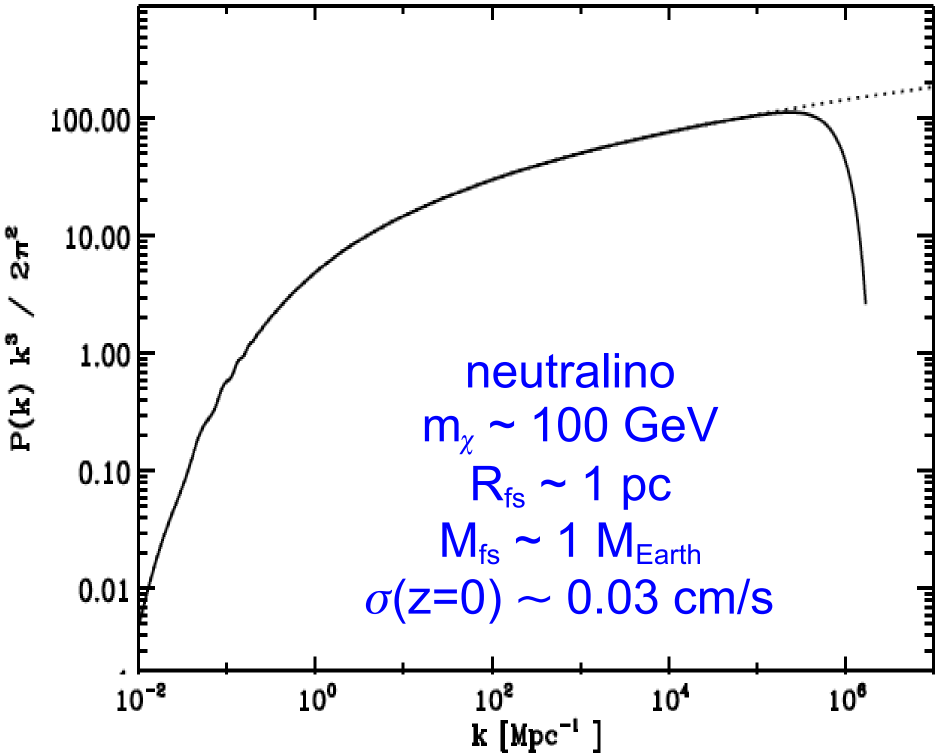

Free streaming length: $R_{\rm fs} \propto m_\chi^{-1}$

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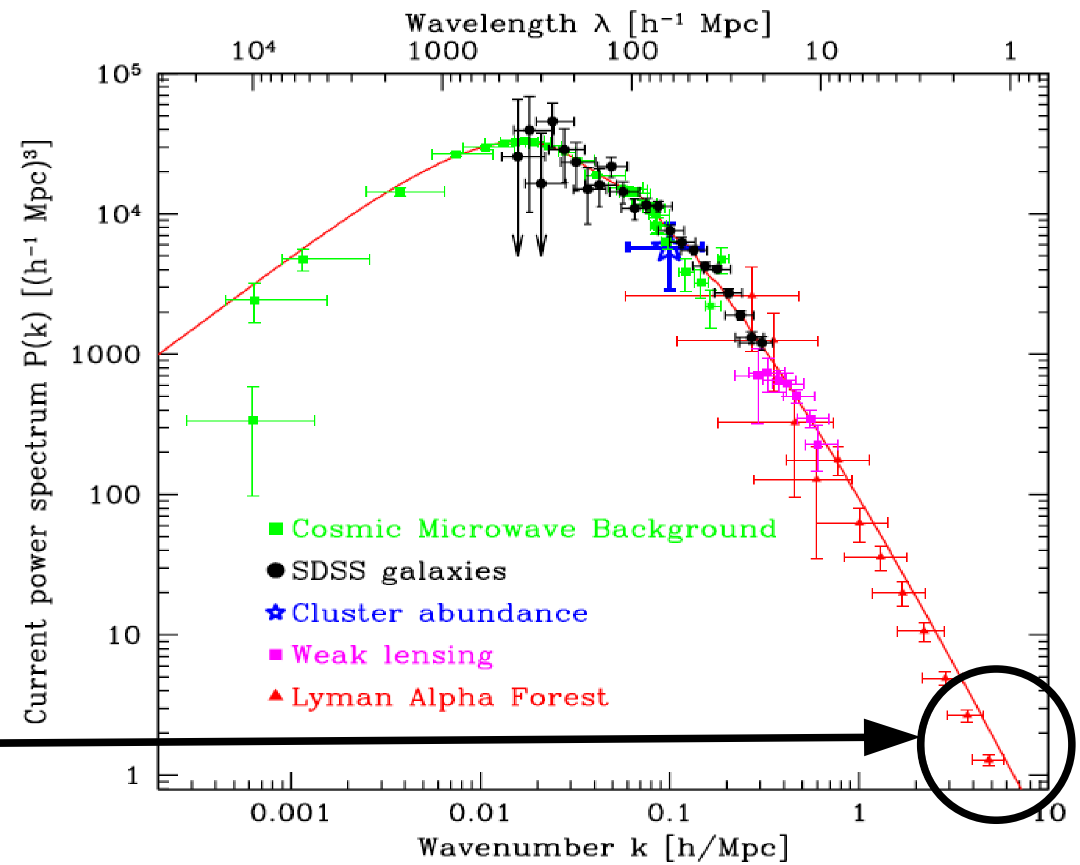
How cold is DM?

e.g. Boyarsky et al. 2009

m_χ > 1 keV
R_{fs} < 500 kpc
M_{fs} < 10^{10} M_{Sun}
$\sigma(z=0) < 0.04$ km/s



Tegmark et al. 2004

The CDM paradigm: DM is collision-less

- In order to generate overdense structures today (Δ>>1), the initial density perturbations need to be much larger than the fluctuations inferred from the CMB. A simple way to accomplish this is to assume that DM decouples from the radiation field long before recombination, essentially behaving as a collision-less fluid afterwards.

- This implies that the phase-space density is conserved after decoupling.

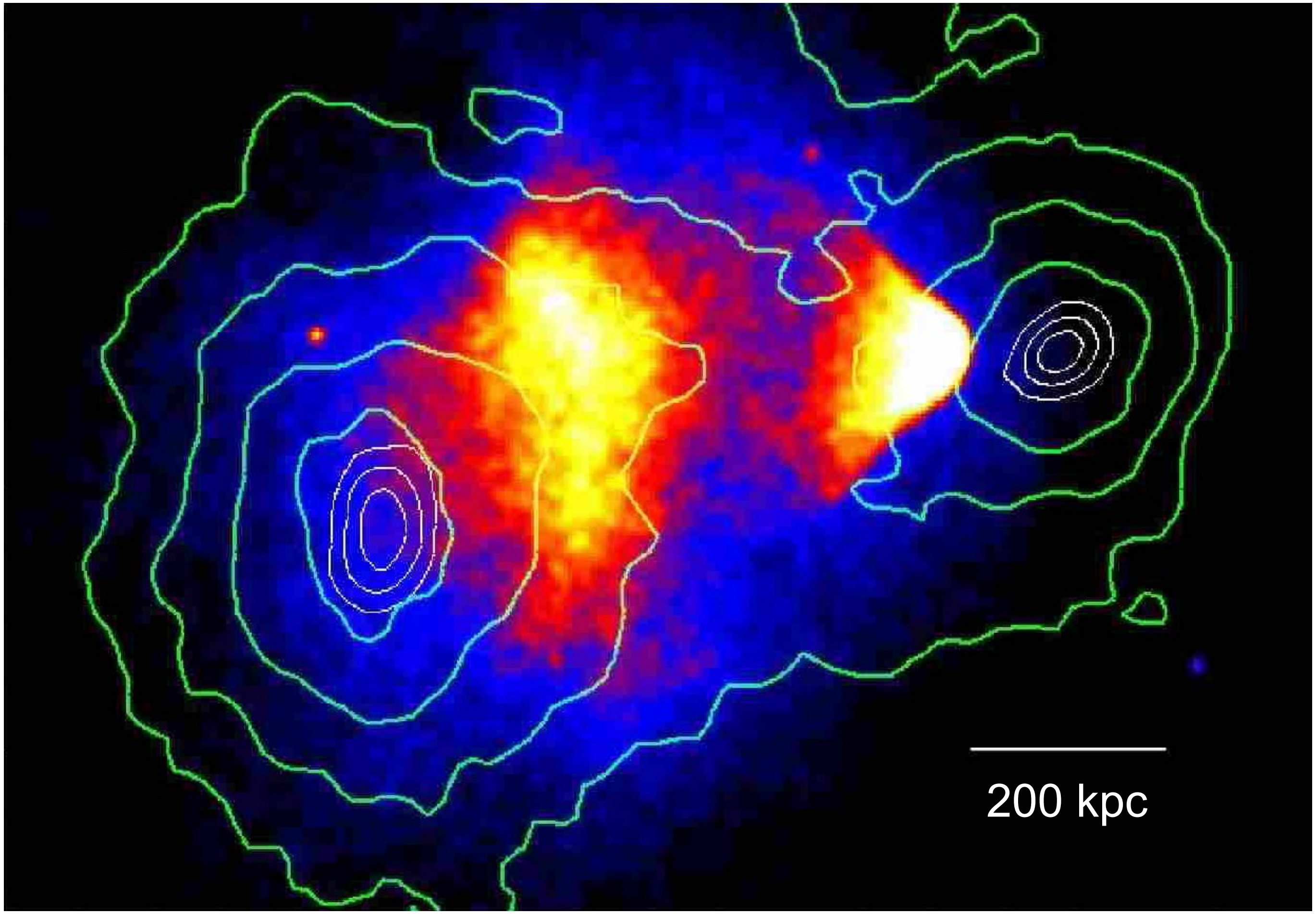
- **How collision-less DM is**, and at what scales, ultimately depends on observational constraints

The CDM paradigm: DM is collision-less

- Assumption: DM behaves as a collision-less fluid after decoupling (the phase-space density is conserved).

- **How collision-less DM is**, and at what scales, ultimately depends on observations

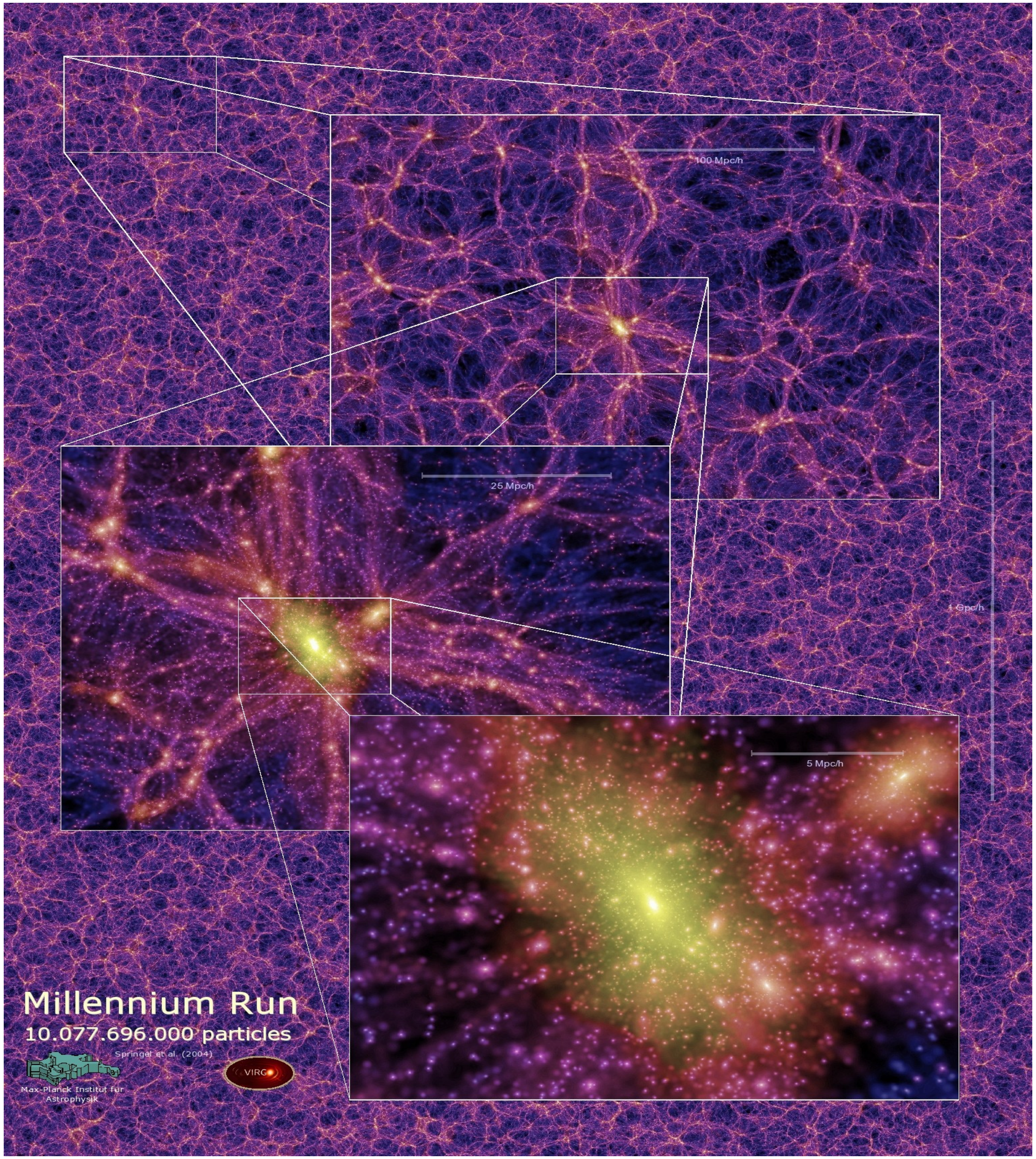
Bullet Cluster (Clowe et al. 2006)



σ / m < 1.25 cm^2/g (for relative velocities of O(1000km/s) Randall et al. 2008)

There are other, stronger, constraints that will be discussed in the last lecture

The large-scale structure of the Universe: confronting the CDM model



The CDM model predicts a large-scale structure with filaments and voids

The large-scale structure of the Universe: confronting the CDM model

The large-scale distribution of the Universe is reproduced by the CDM model!!

Scale ~400-600 Mpc
~ size of Millennium simulation box

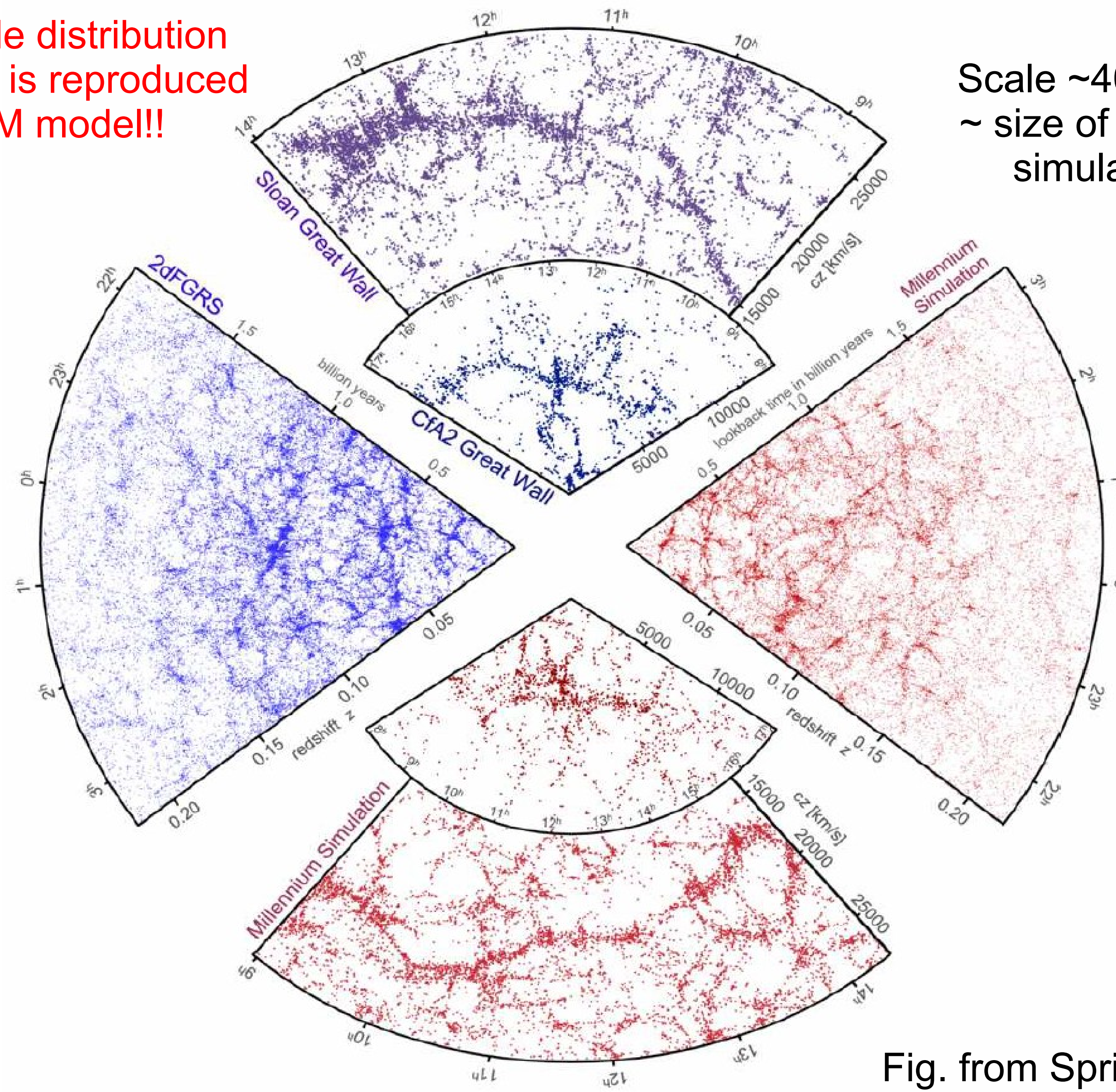


Fig. from Springel et al. 2006

Formation an evolution of DM halos

DM Halo: gravitationally self-bound DM structure.

Halo boundary (mass): radius where the halo has a certain average density contrast: $<\rho_h> = \Delta\rho_{crit}$, this defines a characteristic mass:

$$M = \frac{4}{3}\pi\Delta\,\rho_{\rm crit}\,r_{\rm lim}^3$$

The value of Δ is somewhat conventional, a common choice Is 200, another one is based on the spherical collapse model $\Delta \sim 178\Omega_m^{0.45}$ (~95 at z=0); in both cases it is common to refer to the mass within the boundary as the virial mass.

In the CDM model, structures grow hierarchically: halo mergers are common events

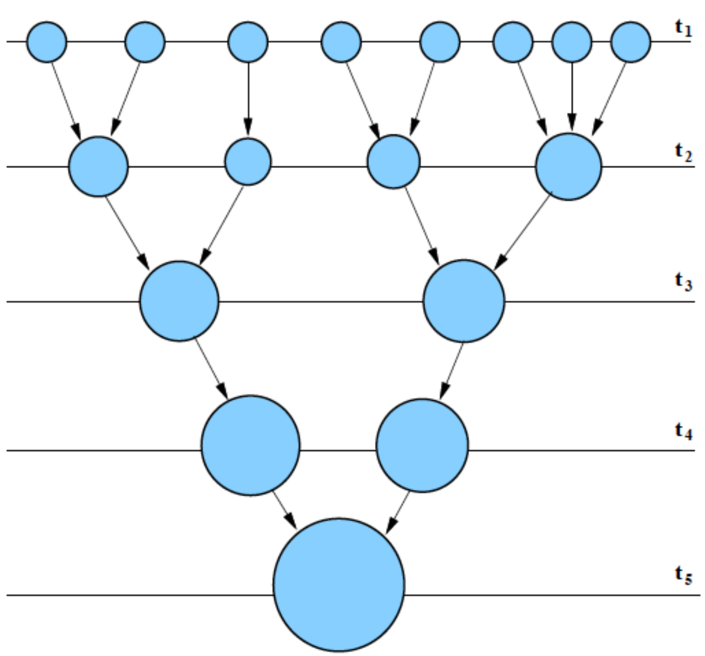
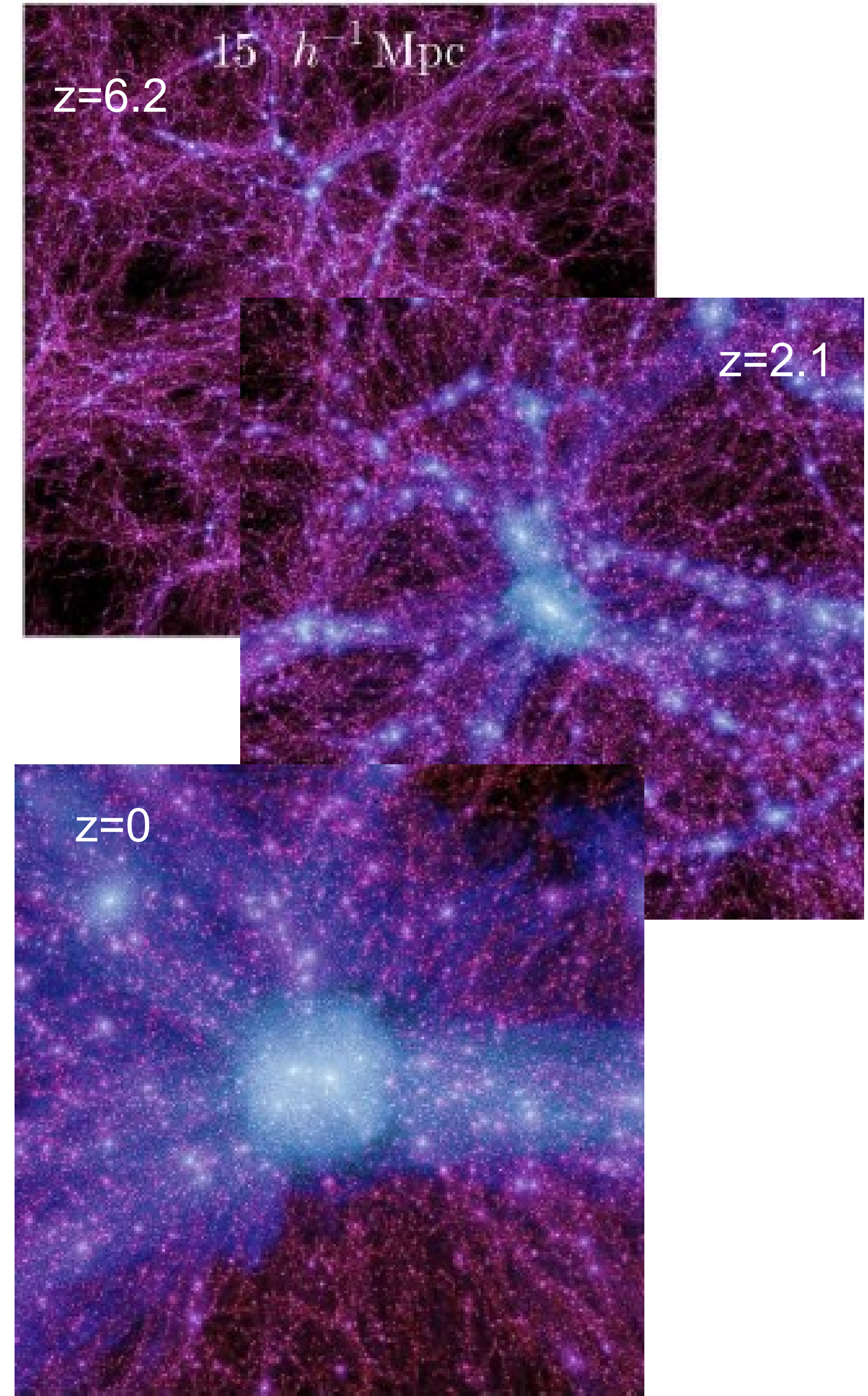


Fig. from Baugh 2006



Figs. from Boylan-Kolchin et al. 2009

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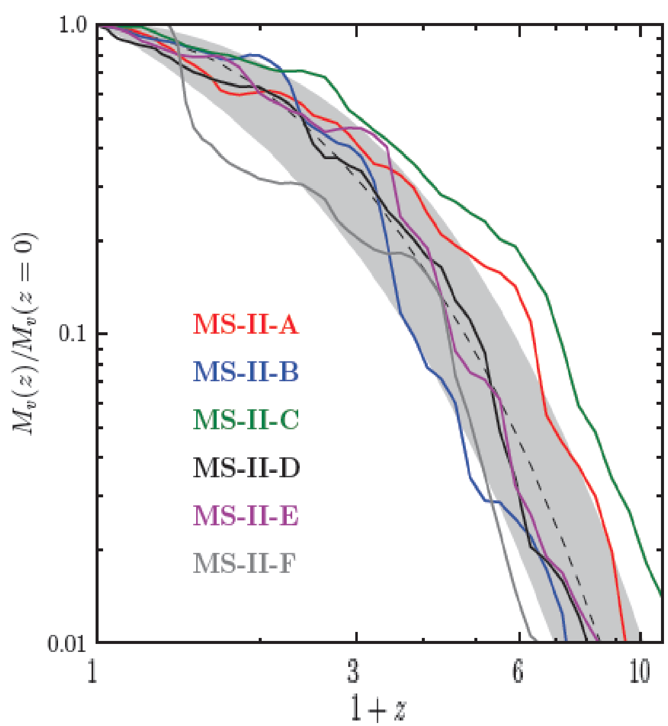
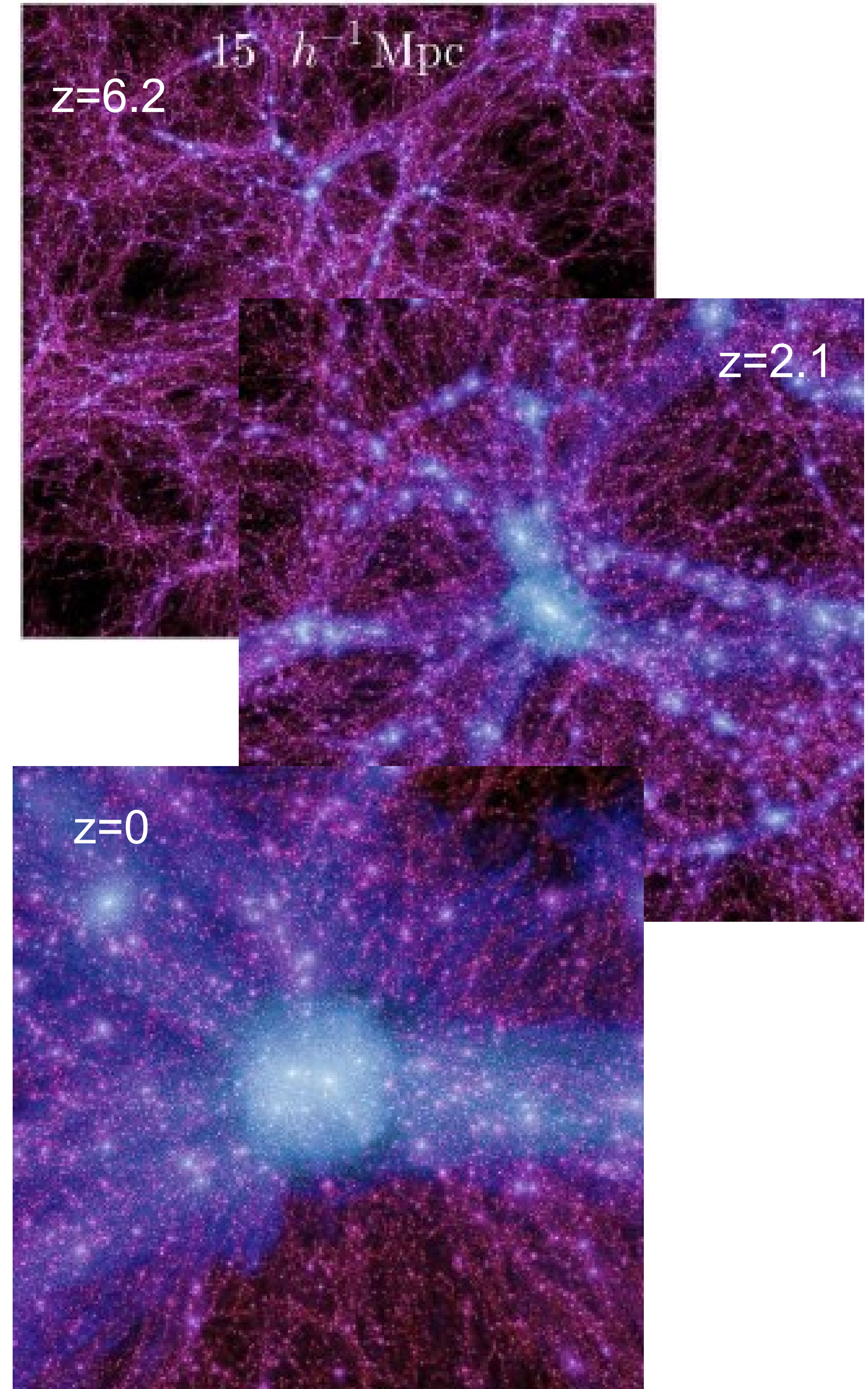
Typical "mass aggregation histories" for MW-size halos.



Boylan-Kolchin et al. 2010



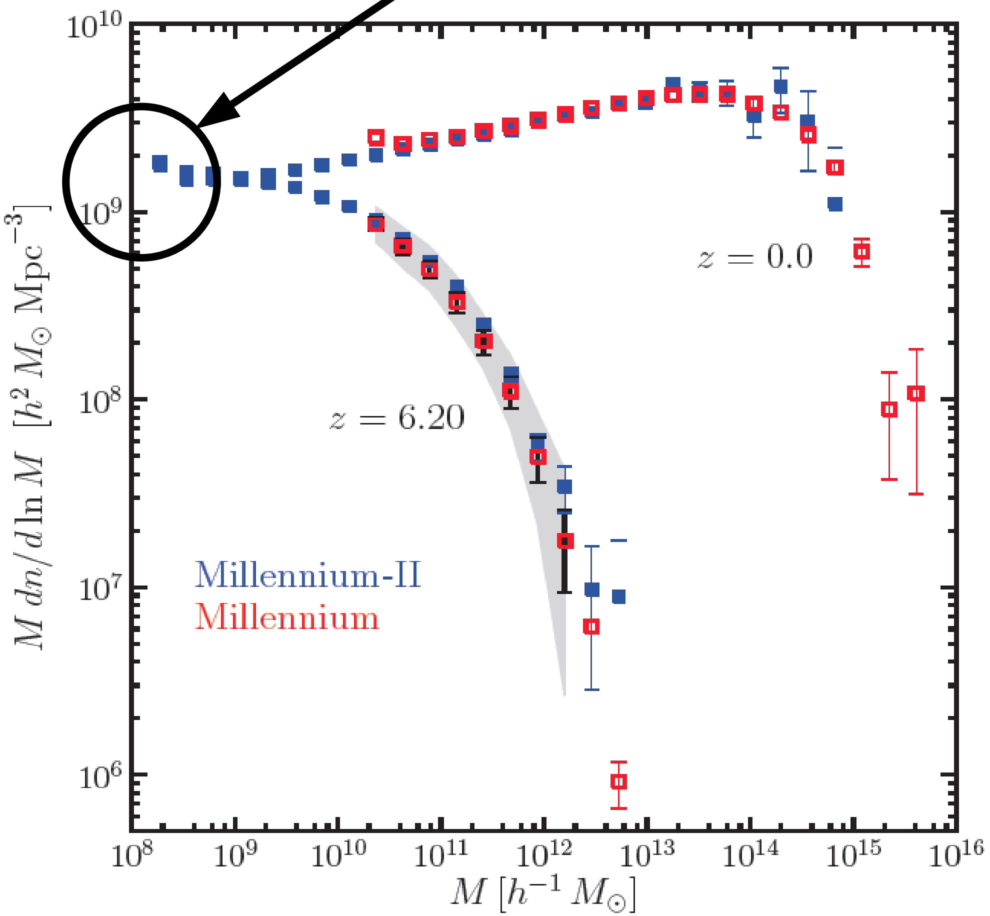
Figs. from Boylan-Kolchin et al. 2009

Predictions of CDM: The abundance of DM halos

Mass function (dn/dM): number of halos per comoving volume and per mass range.
It evolves with redshift according to the hierarchical scenario.

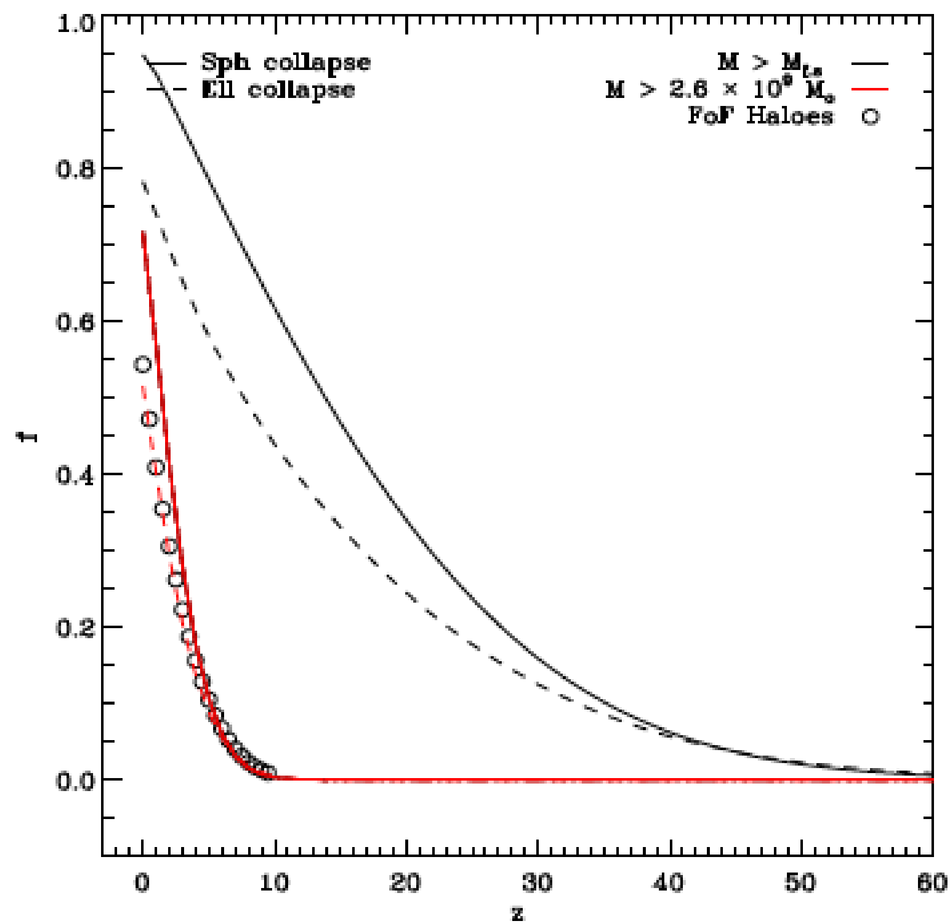
dn/dM ~ M^{-2} (at small masses)

Boylan-Kolchin et al. 2010



CDM free streaming length many orders of magnitude below mass resolution of current simulations!

Angulo and White 2010



Not all dark matter is in halos!

Predictions of CDM: The universal density profile of DM halos

DM halos are not spherical but triaxial. Their internal structure (once in a relaxed state) is however roughly represented by a spherically symmetric radial distribution with a nearly universal profile:

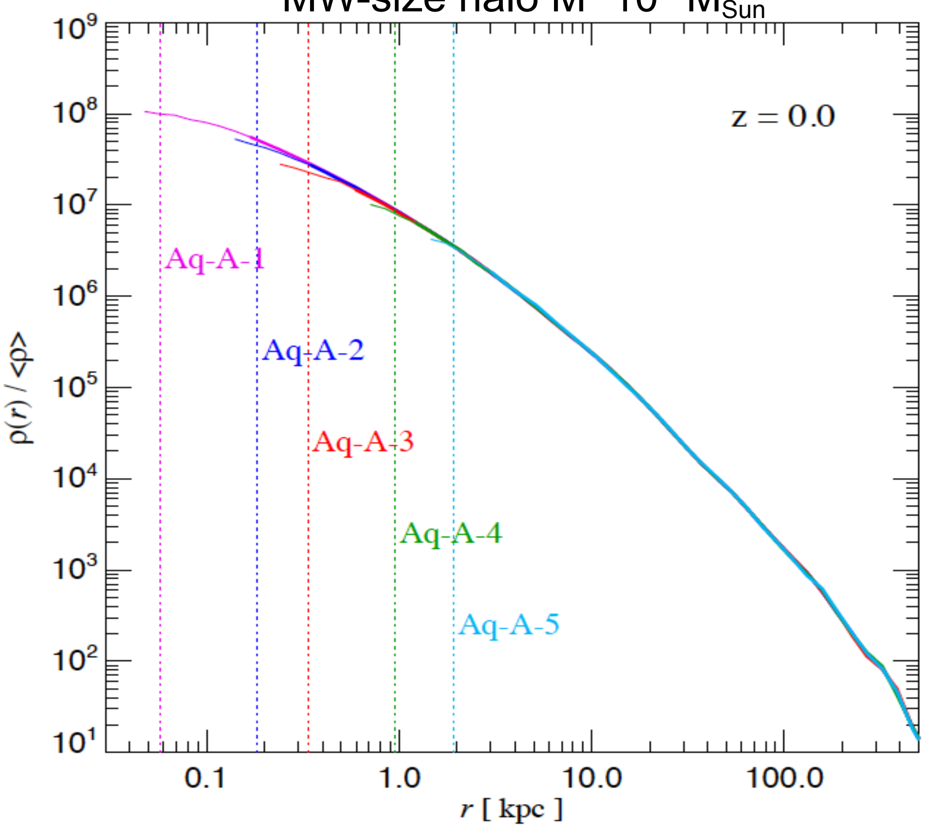
limiting radius

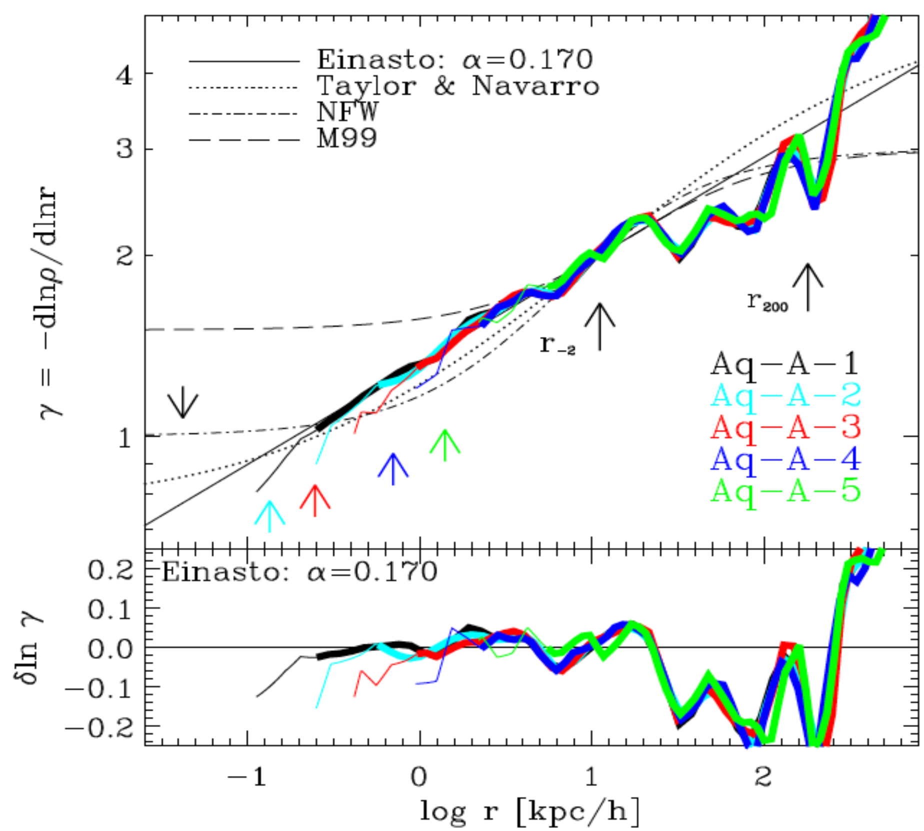
$$\rho(r) = \rho_{crit} \frac{\delta_0}{(r/r_s)(1 + r/r_s)^2} \qquad \delta_0 = \frac{\Delta}{3} \frac{c^3}{\ln(1+c) - c/(1+c)} \qquad c = r_v/r_s$$

This distribution was originally proposed by Navarro, Frenk and White (1996,1997) to fit the results of numerical simulations; it is known as the NFW profile. Recent simulations favour a slightly different profile near the centre (Einasto profile)

MW-size halo M~$10^{12} M_{Sun}$

Springel et al. 2008





Navarro et al. 2010

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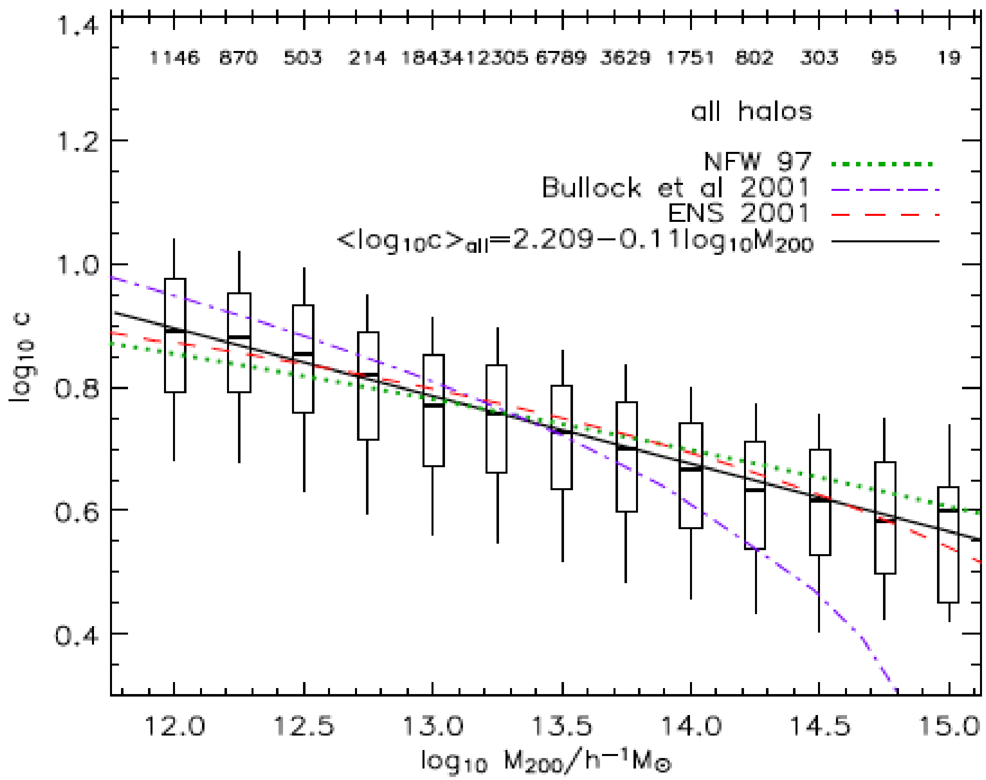
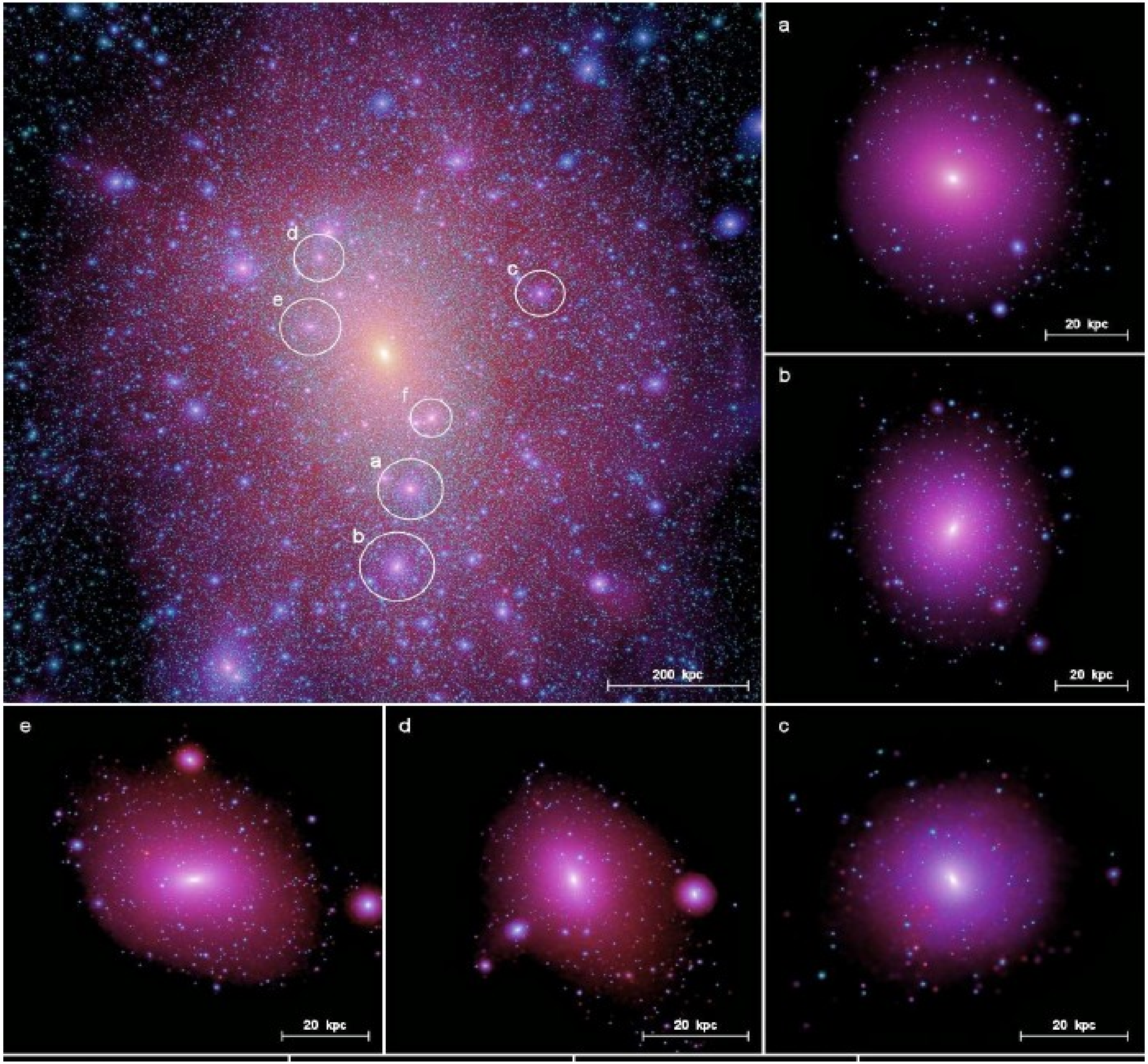
The parameter 'c' is a measure of how densely concentrated a halo is, it reflects the “formation time” of the halo and hence, it is mass-dependent: low-mass halos (forming earlier than more massive ones) have larger concentrations.



Concentration-mass relation Neto et al. 2007

Predictions of CDM: The abundance of substructure

MW- size halo and subhalos (Springel et al. 2008)



Predictions of CDM: The abundance of substructure

MW- size halo and subhalos (Springel et al. 2008)