Gauss' law (Chapter 6)

- We have seen Coulomb's law + principle of superposition (basis of Electrostatics). Three methods to solve problems:
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  - Symmetry and integral Calculus (continuous charge distributions)

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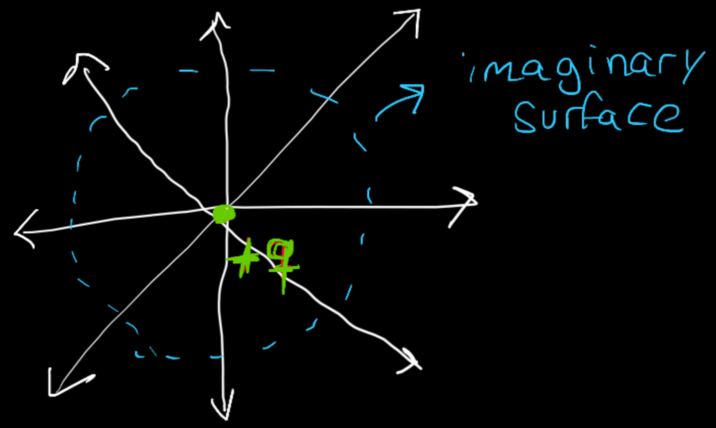
Gauss’ law (Chapter 6)

- **Gauss´ law (discrete or continuous charge distributions)**

$\boxed{\text{Electric Flux}}$

$\vec{E}_q$

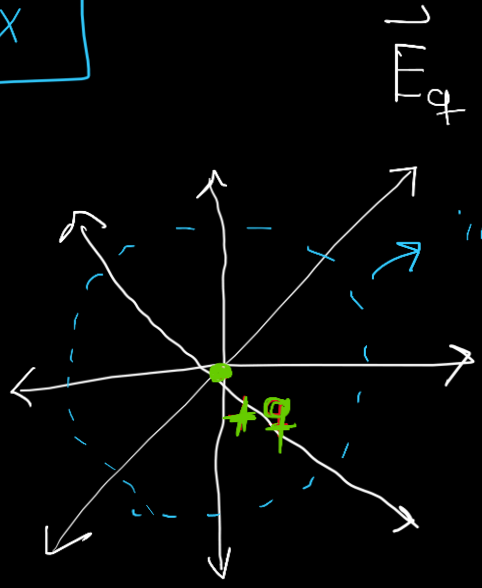


related to the charge +q → The intensity/strength of the electric field is proportional to how many lines cross an imaginary surface

Gauss’ law (Chapter 6)

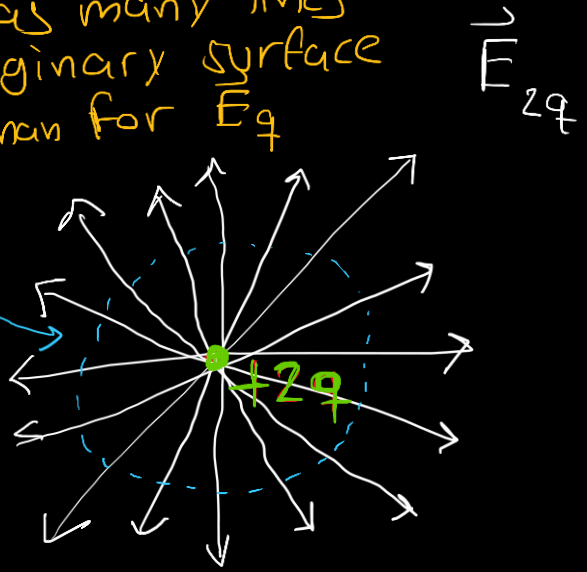
➢ Gauss´ law (discrete or continuous charge distributions)

$\boxed{\text{Electric Flux}}$



imaginary surface

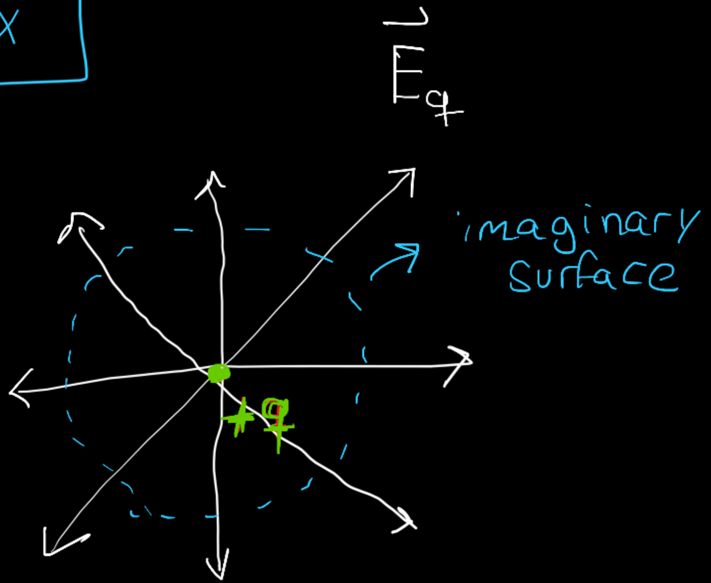
Relatively speaking:
we have twice as many lines crossing the imaginary surface for $\vec{E}_{2q}$ than for $\vec{E}_q$



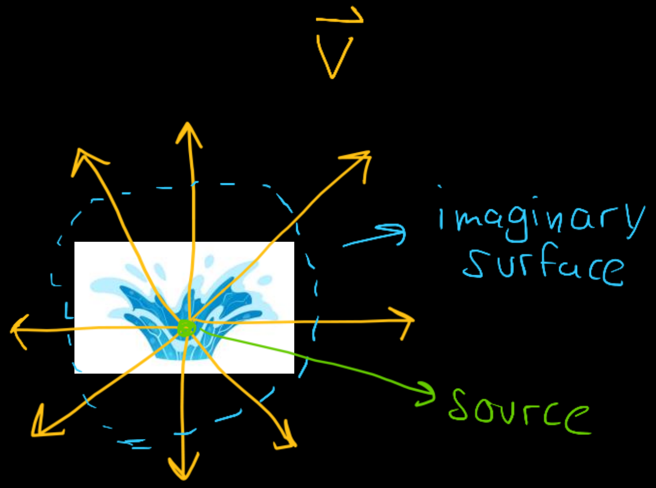
Gauss' law (Chapter 6)

➢ **Gauss´ law (discrete or continuous charge distributions)**

$\boxed{\text{Electric Flux}}$



Water flow analogy



The intensity/strength of the electric field is proportional to how many lines cross an imaginary surface

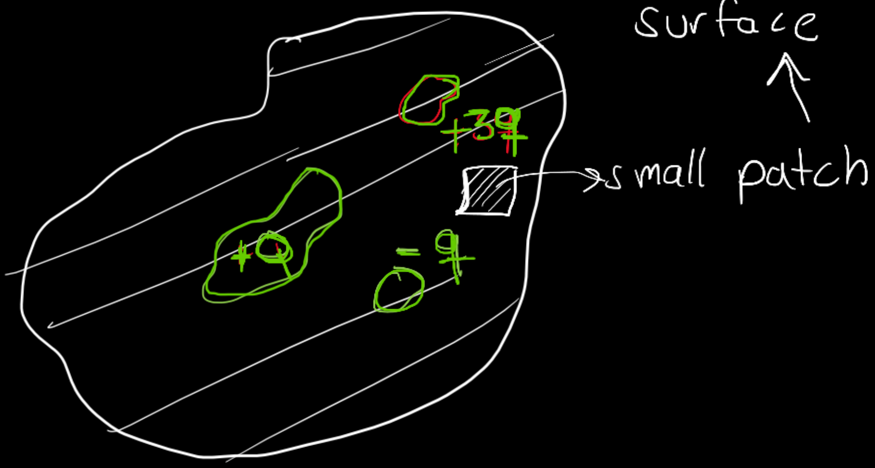
related → to the charge +q

$\vec{V}$ = velocity vector field

We can figure out the rate at which water is produced at the source by counting the lines that cross an imaginary surface

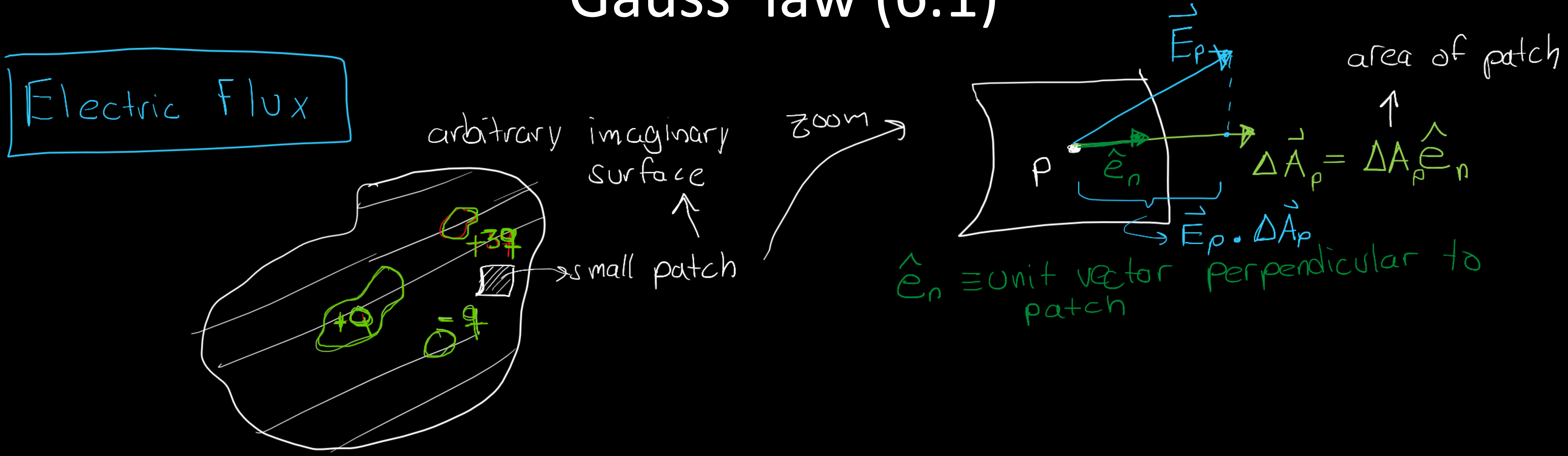
Gauss' law (6.1)

Electric Flux



→ We want to quantify the intensity/strength of the total electric field by "counting" the number of electric field lines passing through the surface

Gauss' law (6.1)

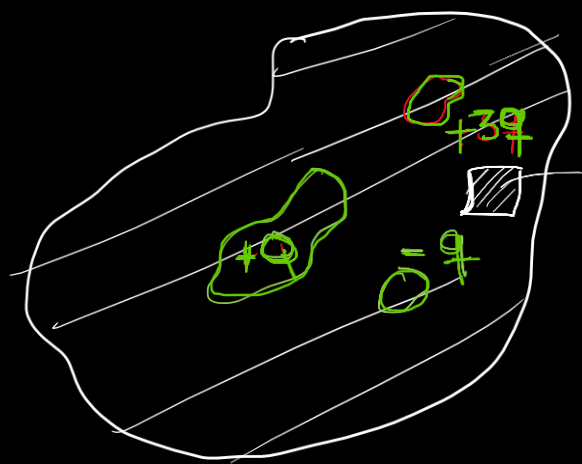

Electric Flux
arbitrary imaginary surface
small patch
zoom
+3q
+q
-q
p
$\hat{e}_n$
$\vec{E}_p$
$\vec{E}_p \cdot \Delta\vec{A}_p$
area of patch
$\Delta\vec{A}_p = \Delta A_p \hat{e}_n$
$\hat{e}_n \equiv$ unit vector perpendicular to patch

Gauss' law (6.1)

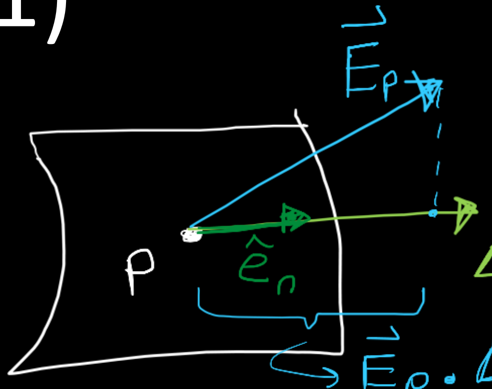
Electric Flux

arbitrary imaginary surface

small patch



zoom



area of patch

$\Delta \vec{A}_p = \Delta A_p \hat{e}_n$

$\vec{E}_p \cdot \Delta \vec{A}_p$

$\hat{e}_n \equiv$ unit vector perpendicular to patch

dot/scalar product

$$\Delta \phi_p = \vec{E}_p \cdot \Delta \vec{A}_p = \underbrace{(\vec{E}_p \cdot \hat{e}_n)}_{\text{component of } \vec{E}_p \text{ along } \hat{e}_n} \Delta A_p$$

electric flux
$\equiv$
amount of electric field lines piercing through the surface

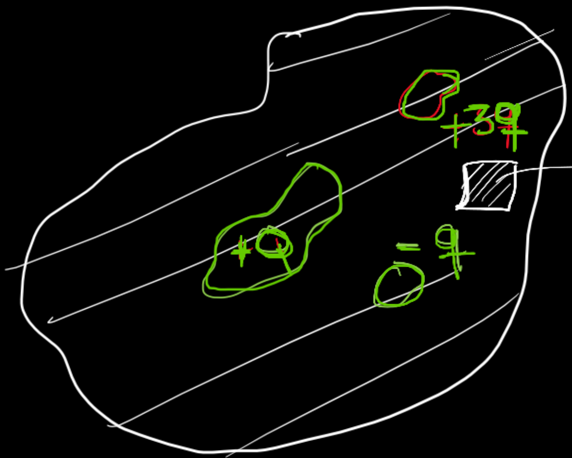
component of $\vec{E}_p$ along $\hat{e}_n$

only this one contributes to the flux

Gauss' law (6.1)

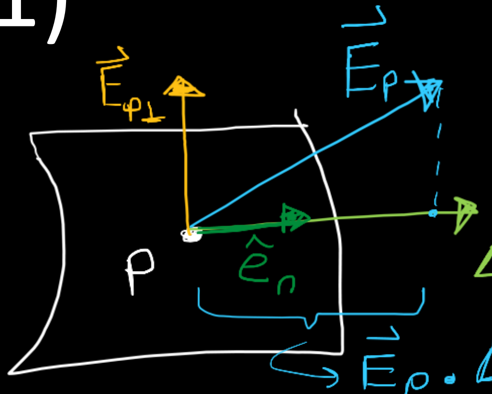
Electric Flux

arbitrary imaginary surface



small patch

zoom



area of patch

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electric flux

"

amount of electric field lines piercing through the surface

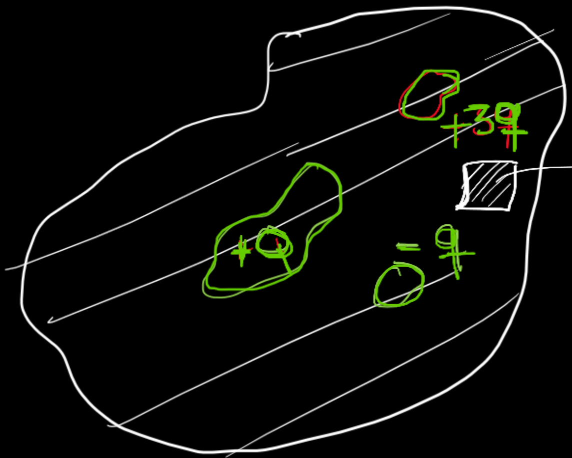
component of $\vec{E}_p$ along $\hat{e}_n$

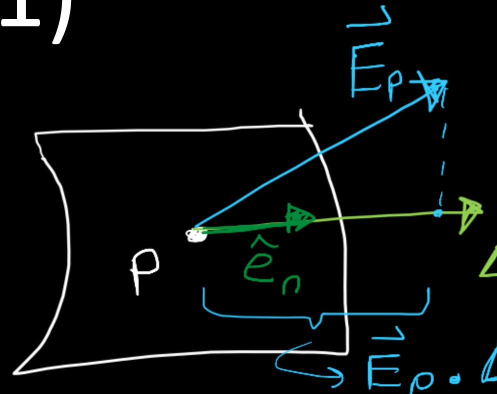
only this one contributes to the flux

The component of $\vec{E}_p$ perpendicular to patch: $\vec{E}_{p\perp}$ doesn't contribute to the flux (it doesn't pierce/passes through the patch)

Gauss' law (6.1)

Electric Flux





area of patch

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$\hat{e}_n \equiv$ unit vector perpendicular to patch

dot/scalar product

$$\Delta\phi_p = \vec{E}_p \cdot \Delta\vec{A}_p = \underbrace{(\vec{E}_p \cdot \hat{e}_n)}_{\text{component of } \vec{E}_p \text{ along } \hat{e}_n} \Delta A_p$$

$\Delta\phi_p$: electric flux ≡ amount of electric field lines piercing through the surface

component of $\vec{E}_p$ along $\hat{e}_n$ → only this one contributes to the flux

⟹ Total flux over whole surface:

$$\boxed{\phi = \int_S \vec{E} \cdot d\vec{a}}$$

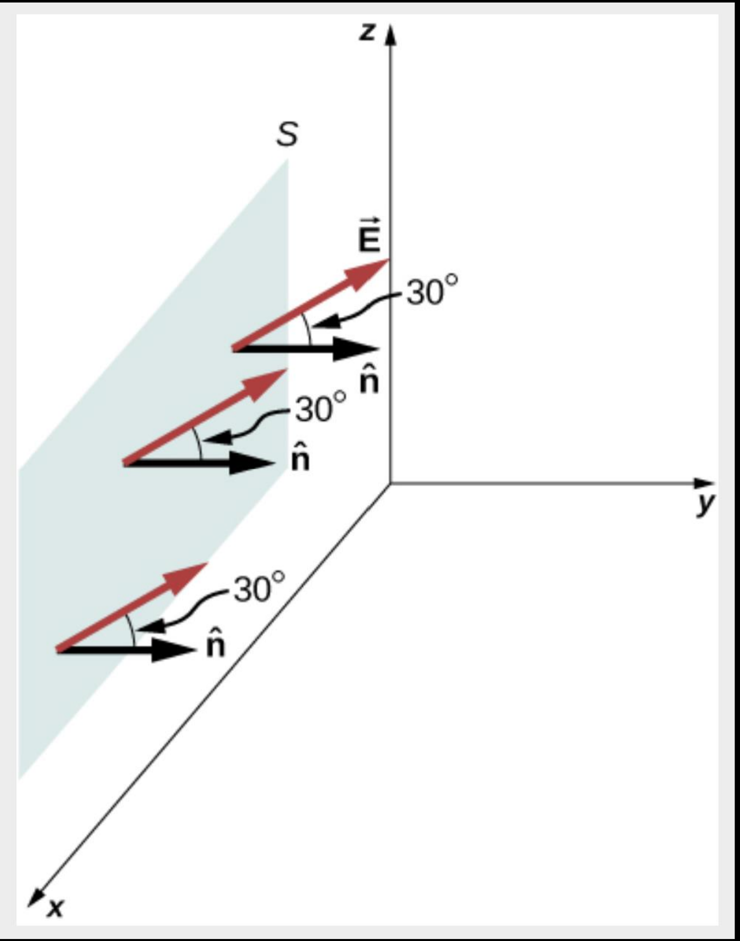
Total flux

$S \equiv$ surface (open or closed)

$\int_S \equiv$ surface integral

Electric flux: example 6.3

Calculate flux through open surface S of area $A = 6\,m^2$ located in xz-plane due to uniform $\vec{E}$ with $|\vec{E}| = 10\,N/C$ ($\vec{E}$ makes an angle $\theta = 30^\circ$ with xz-plane)



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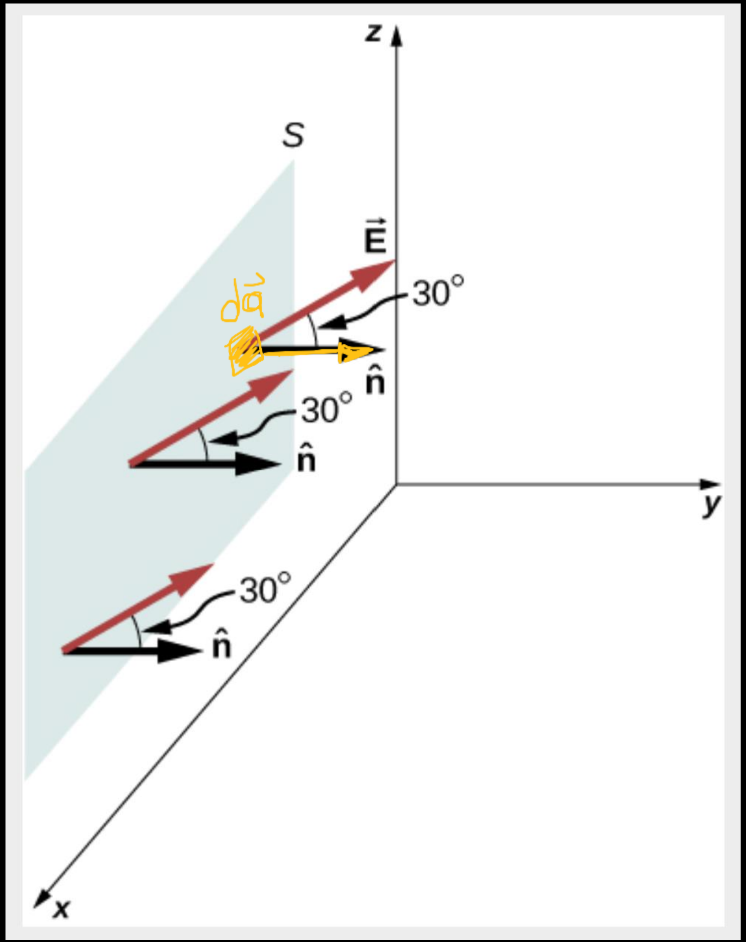

$$\boxed{\phi = \int_S \vec{E} \cdot d\vec{a}}$$ Total flux

$$d\vec{a} = da\,\hat{n}$$

$\hat{e}_n$ (notation)

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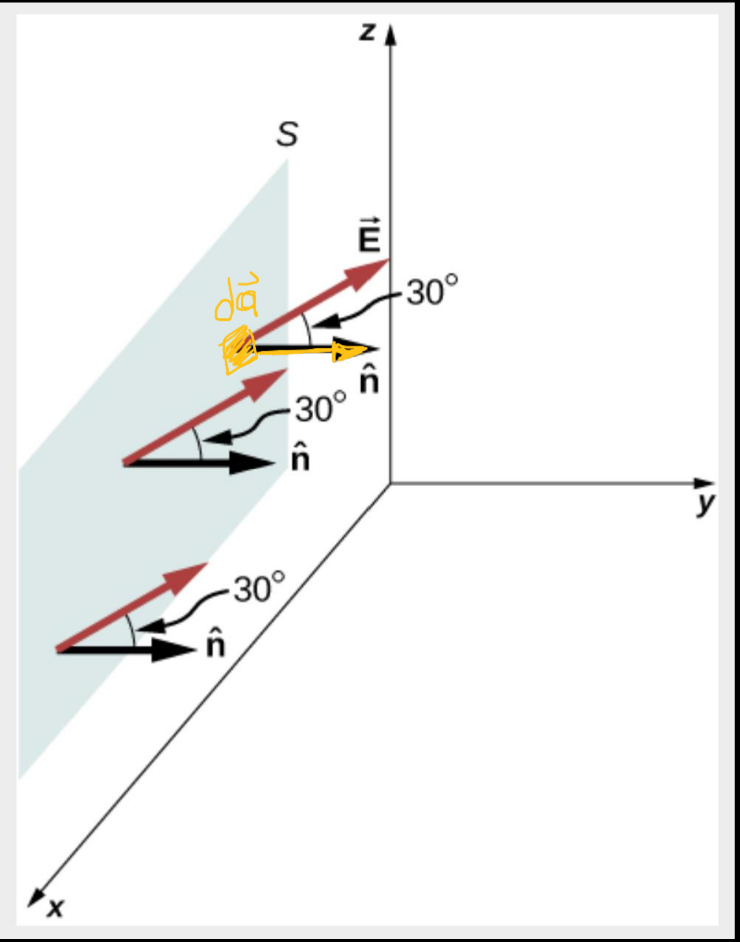

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$$d\vec{a} = da\,\hat{n}$$ $\hat{e}_n$ (notation)

$$\Rightarrow \vec{E} \cdot d\vec{a} = |\vec{E}|\,da\,\cos\theta \qquad (\theta = 30°)$$

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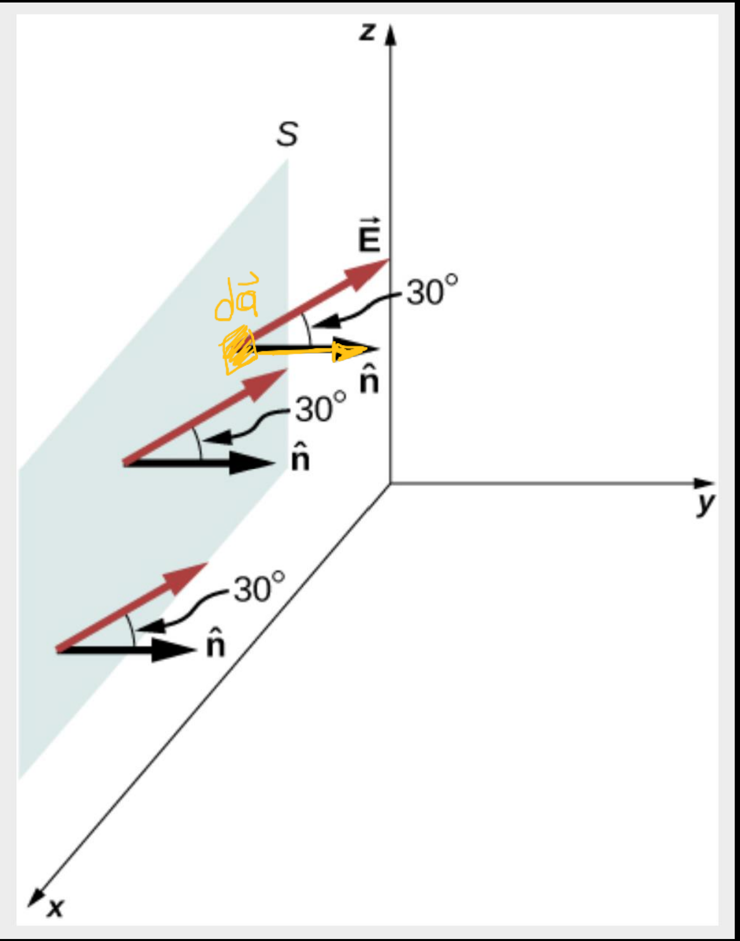

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$\hat{e}_n$ (notation)

$$d\vec{a} = da\,\hat{n}$$

$$\Rightarrow \vec{E} \cdot d\vec{a} = |\vec{E}|\,da\,\cos\theta \qquad (\theta = 30^\circ)$$

$$\Rightarrow \phi = \int_S |\vec{E}|\,da\,\cos\theta$$

$\cos\theta \rightarrow$ constant

$|\vec{E}|$: constant $= 10\ N/C$

Electric flux: example 6.3

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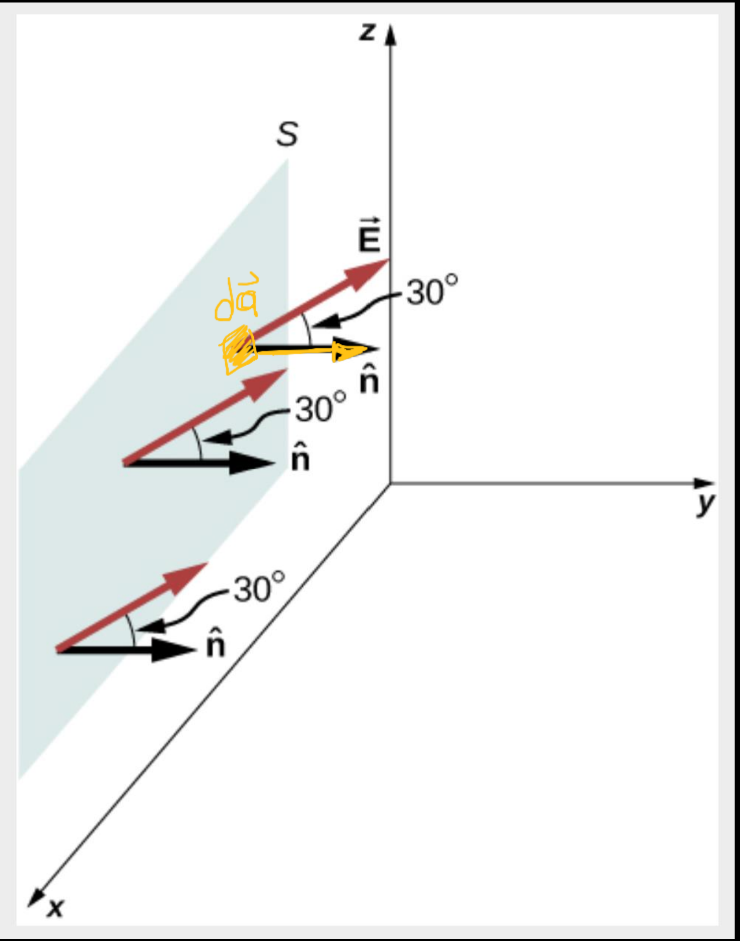


$$\boxed{\phi = \int_S \vec{E} \cdot d\vec{a}}$$ Total flux

$d\vec{a} = da\,\hat{n}$ ← $\hat{e}_n$ (notation)

$\Rightarrow \vec{E} \cdot d\vec{a} = |\vec{E}|\, da \cos\theta$ $\quad(\theta = 30°)$

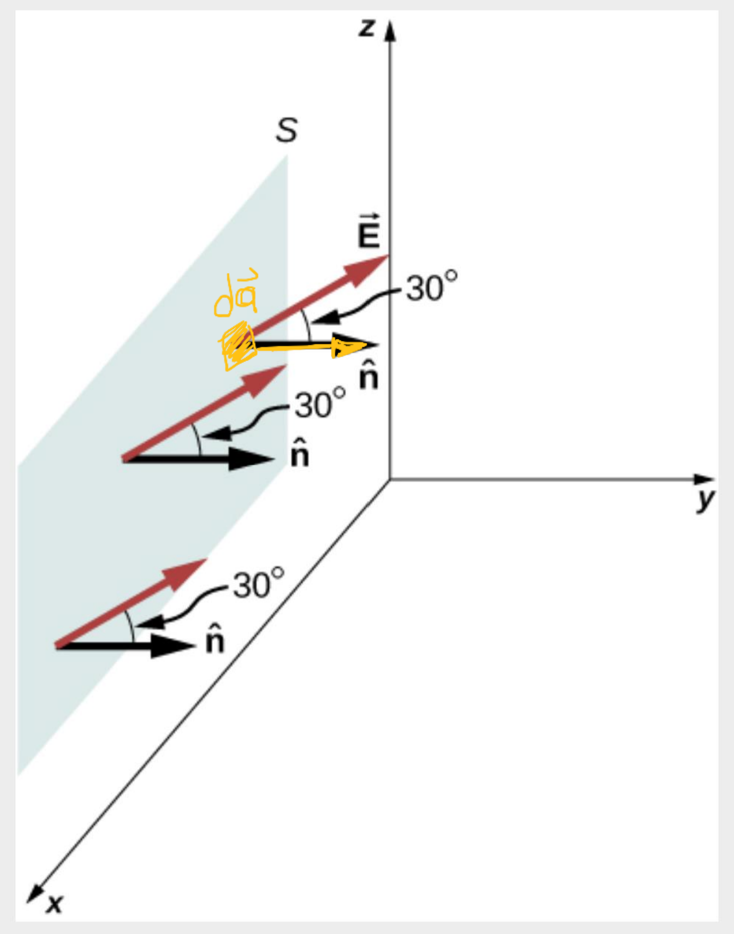
$\Rightarrow \phi = \int_S |\vec{E}|\, da \cos\theta$ → constant

$|\vec{E}|$: constant $= 10\,N/C$

$\Rightarrow \phi = |\vec{E}| \cos\theta \int_S da$ → area of surface $= A = 6\,m^2$

Electric flux: example 6.3

Calculate flux through open surface S of area $A = 6\,m^2$ located in xz-plane due to uniform $\vec{E}$ with $|\vec{E}| = 10\,N/C$ ($\vec{E}$ makes an angle $\theta = 30°$ with xz-plane)



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$$\Rightarrow \vec{E} \cdot d\vec{a} = |\vec{E}|\, da \cos\theta \qquad (\theta = 30°)$$

$$\Rightarrow \phi = \int_S |\vec{E}|\, da \cos\theta$$

$\cos\theta \rightarrow$ constant

$|\vec{E}|$: constant $= 10\,N/C$

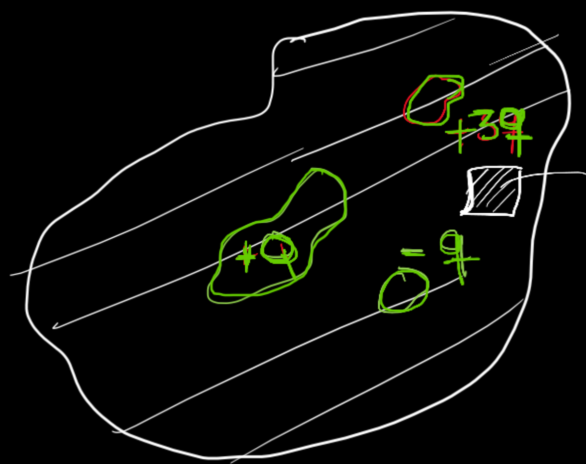
$$\Rightarrow \phi = |\vec{E}| \cos\theta \int_S da = |\vec{E}|\, A \cos\theta$$

$$= (10\,N/C)(6\,m^2)\cos(30°) \sim 52\,\frac{N\,m^2}{C}$$

don't get confused between radians/degrees

Gauss' law (6.1)

Electric Flux



$\Rightarrow$ Total flux over whole

$$\boxed{\Phi = \int_S \vec{E} \cdot d\vec{a}}$$ Total flux

$S \equiv$ surface (open or closed)

$\int_S \equiv$ surface integral

$\rightarrow$ But we know that $\vec{E}$ fills/permeates all space

$\Rightarrow$ we need closed surface to count all lines

$$\boxed{\Phi = \oint_S \vec{E} \cdot d\vec{a}}$$ $\rightarrow$ Net flux over closed surface $\rightarrow$ This is the one more relevant to Electrostatics

Direction: inward piercing field $\Rightarrow$ negative flux

outward " " $\Rightarrow$ positive flux

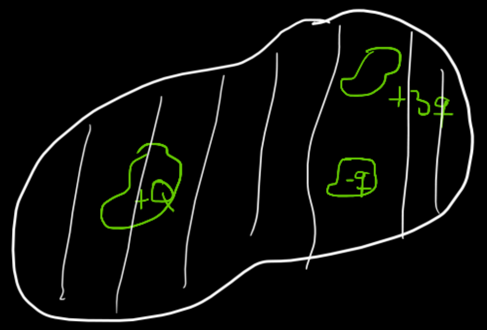
Gauss' law (Chapter 6.2)

Electric Flux

arbitrary imaginary "Gaussian" surface S

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a}}$$

Net flux over closed surface



$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law (*)

Gauss' law relates the net electric flux through a closed surface (Gaussian surface) to the net charge Q_{enc} that is enclosed by the surface

$Q_{enc} \equiv$ total enclosed charge by the surface (Gaussian surface)

$\varepsilon_0 \equiv$ permitivity of vacuum $= 8.85 \times 10^{-12} \frac{C^2}{N\,m^2}$

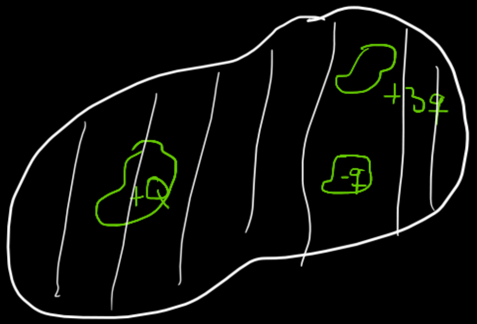
$K = \frac{1}{4\pi\varepsilon_0} \equiv$ Coulomb's constant

(*) Only valid in vacuum

Gauss' law (Chapter 6.2)

Electric Flux

arbitrary imaginary "Gaussian" surface S



$$\boxed{\Phi = \oint_S \vec{E} \cdot d\vec{a}}$$

Net flux over closed surface

$$\boxed{\Phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law (*)

Notes:

- The equation is powerful/elegant: the total flux, no matter the surface, only depends on the total enclosed charge, no matter how complicated its distribution is

- Charges outside the surface do not contribute to the total enclosed charge. The electric field however, does contain contributions from all charges (inside and outside the surface).

Gauss' law

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law
(*)

→ Gauss' law ≡ Coulomb's law

Gauss' law

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law (*)

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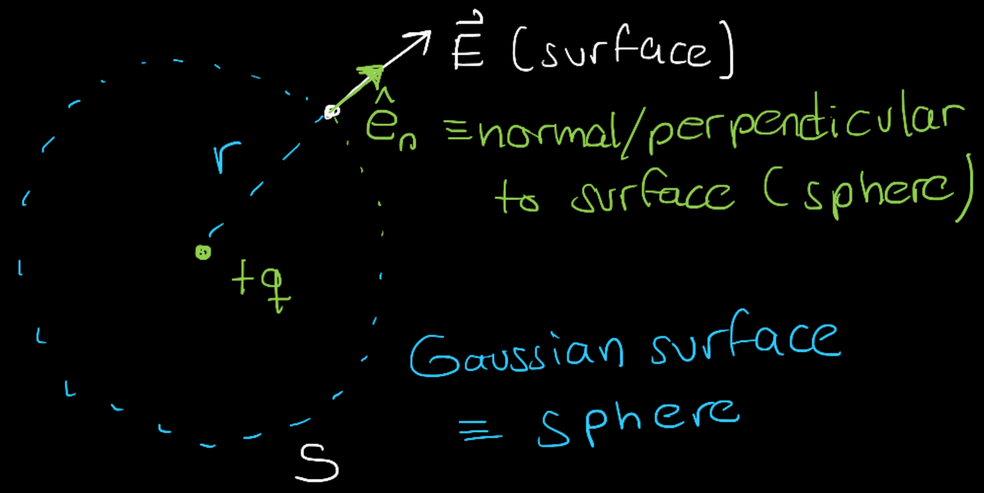


→ Clearly: $\vec{E} \cdot \hat{e}_n = |\vec{E}|$

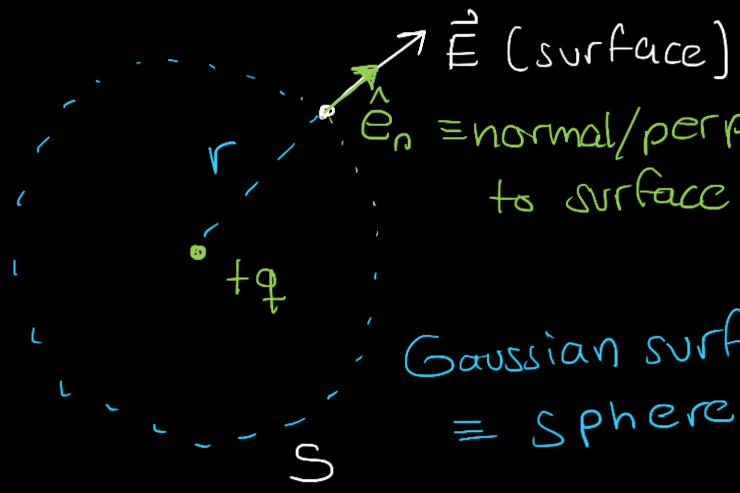
$\vec{E} \cdot d\vec{a} = |\vec{E}| \, da$ $\qquad d\vec{a} = da \, \hat{e}_n$

Gauss' law

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law (*)

$\rightarrow$ Gauss' law $\equiv$ Coulomb's law



$\hat{e}_n \equiv$ normal/perpendicular to surface (sphere)

Gaussian surface $\equiv$ sphere

$\rightarrow$ Clearly: $\vec{E} \cdot \hat{e}_n = |\vec{E}|$

$$\vec{E} \cdot d\vec{a} = |\vec{E}|\, da \qquad d\vec{a} = da\, \hat{e}_n$$

$\rightarrow$ Apply Gauss' law:

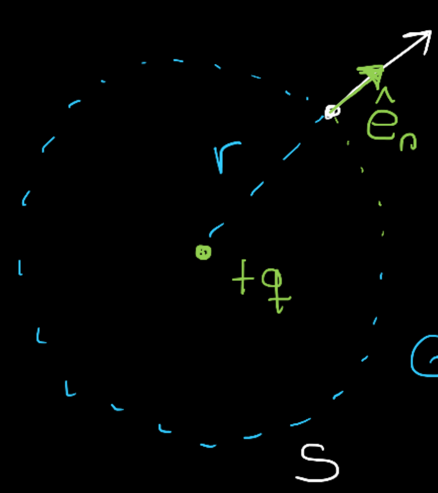
$$\frac{Q_{enc}}{\varepsilon_0} = \frac{+q}{\varepsilon_0} = \oint_S \vec{E} \cdot d\vec{a} = \oint_S |\vec{E}|\, da$$

Gauss' law

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$$

Gauss' law (*)

→ Gauss' law ≡ Coulomb's law



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→ Apply Gauss' law:

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{+q}{\varepsilon_0} = \oint_S \vec{E} \cdot d\vec{a} = \oint_S |\vec{E}| da$$

→ Since $|\vec{E}|$ is the same across all points in the surface (symmetry)

$$\Rightarrow \frac{+q}{\varepsilon_0} = |\vec{E}| \underbrace{\oint_S da}_{\text{Total area of surface}} = |\vec{E}| (4\pi r^2) \Rightarrow \underline{\underline{|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}}$$

→ Coulomb's law

Gauss' law (Chapter 6.4)

Conductors: materials through which charges can move freely (e.g. metals, water, human body)

- A charged conductor (isolated) has a charge that is distributed in a way to satisfy the electrostatic equilibrium condition (after a while, once there is no current/charge-motion anymore).

Non-conductors (insulators): materials through which charges cannot move freely (rubber, plastic)

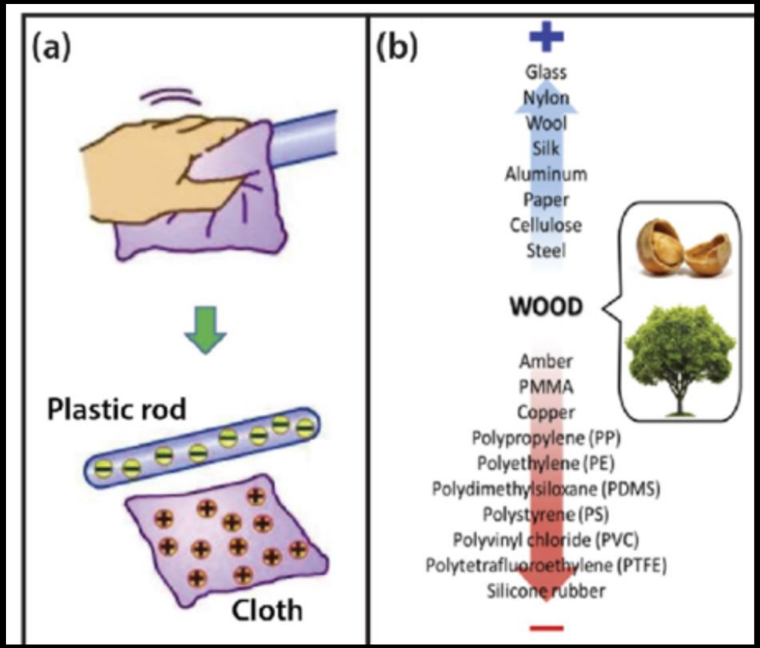
If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the charge will be found within the body of the conductor.

How to charge materials? (Chapter 5.2)

Insulators

→ electrons are tightly bound to atoms

⇒ charging occurs to friction across surfaces removing electrons from one material into another (triboelectric effect)



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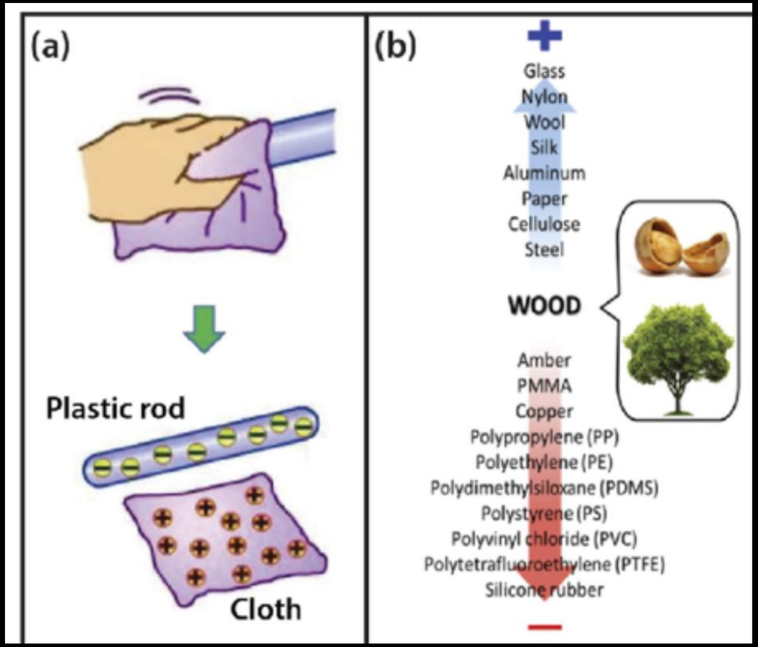
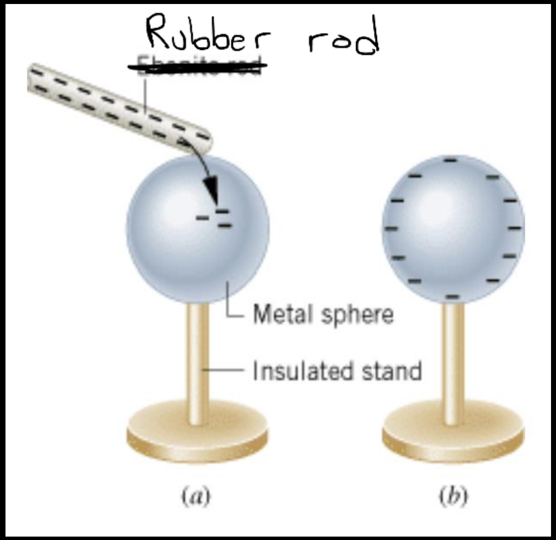


Conductors

→ electrons can move freely

⇒ charging can occur directly by contact through conduction



How to charge materials? (Chapter 5.2)

Insulators

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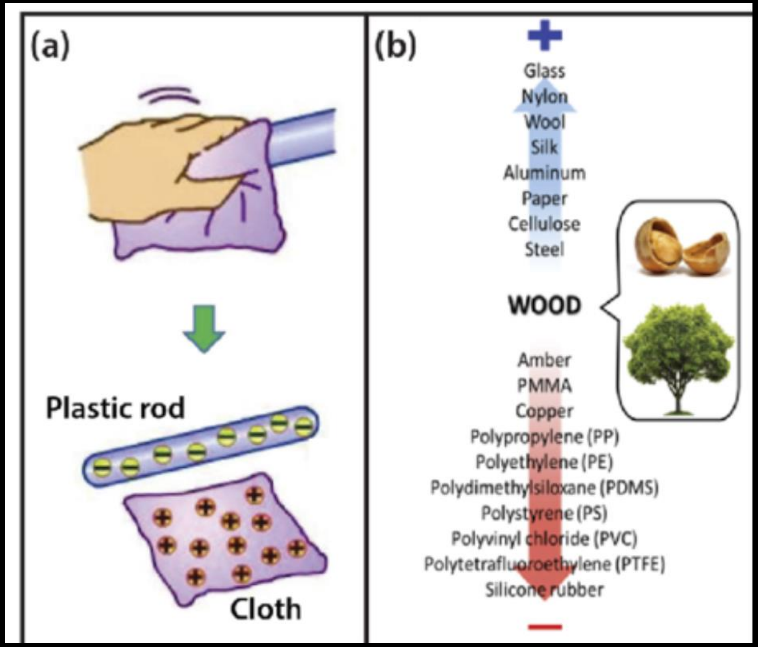
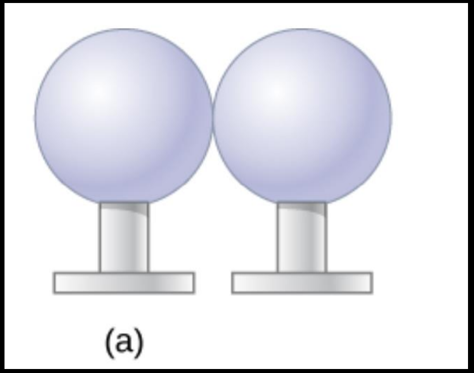


Conductors

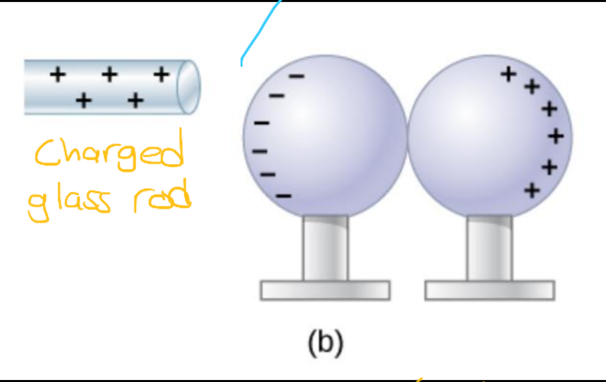
→ electrons can move freely

⇒ charging can occur indirectly by <u>induction</u>

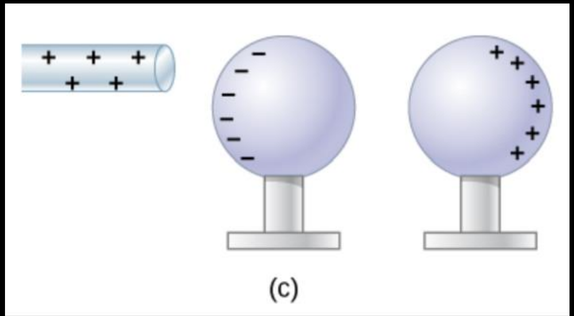
No contact



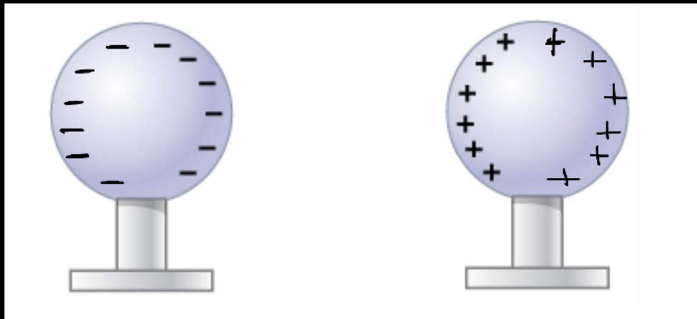
2 <u>neutral</u> conducting spheres



charges separate (polarization)



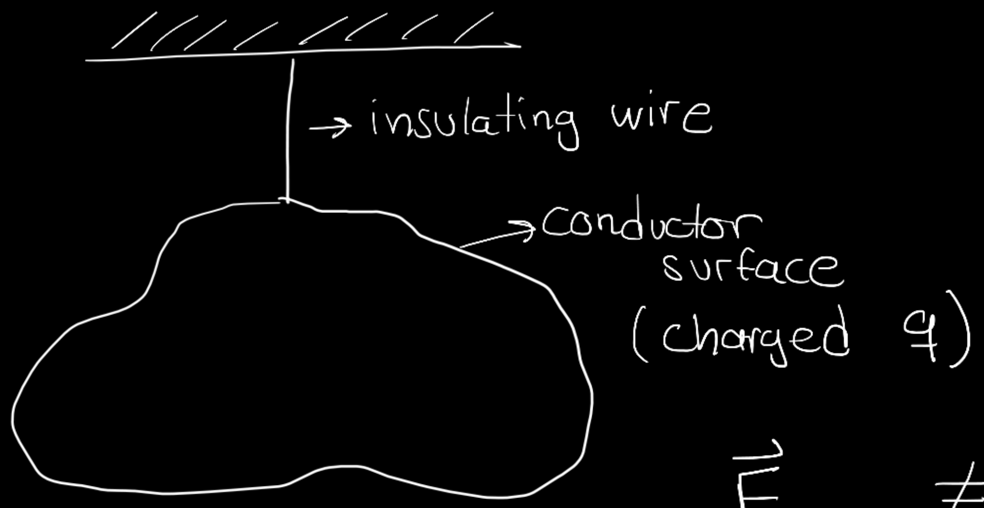
the 2 spheres are separated



Negatively Charged

positively Charged

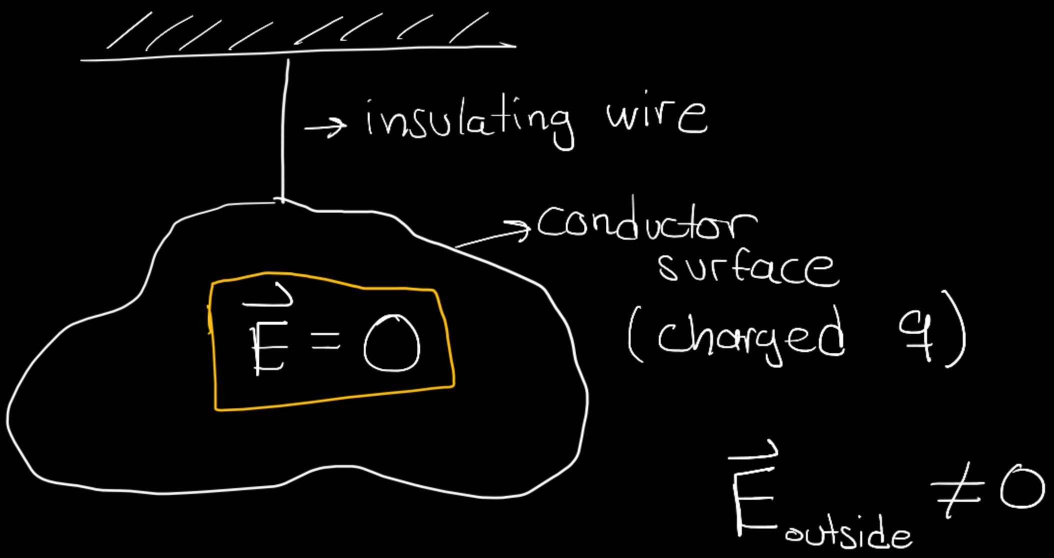
Gauss' law: conductors (Chapter 6.4)



$\vec{E}_{outside} \neq 0$

→ Inside the conductor ?

Gauss' law: conductors (Chapter 6.4)



→ Inside the conductor?

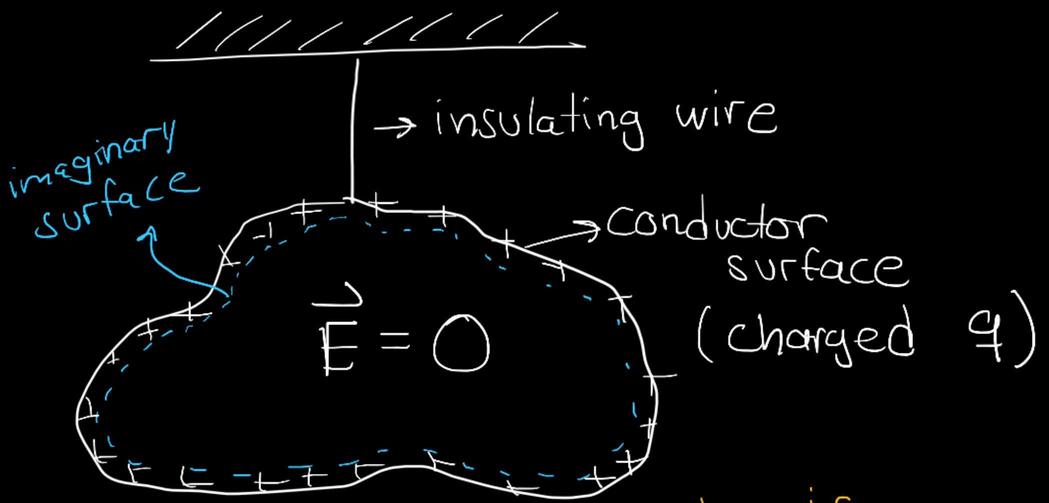
$\boxed{\vec{E} = 0}$

if $\vec{E} \neq 0$ ⇒ electic current because charges are <u>free</u> to move in a conductor

⇒ electrostatic equilibrium condition is not satisfied

→ In reality charges (electrons) in a conductor take a (very short) time to move (during the charging process) to satisfy the electrostatic equilibrium condition

Gauss' law conductors (Chapter 6.4)



* In this case the conductor is positively charged as an example, but it can also be negatively charged

→ Inside the conductor?

$\vec{E} = 0$

if $\vec{E} \neq 0$ ⇒ electic current because charges are free to move in a conductor

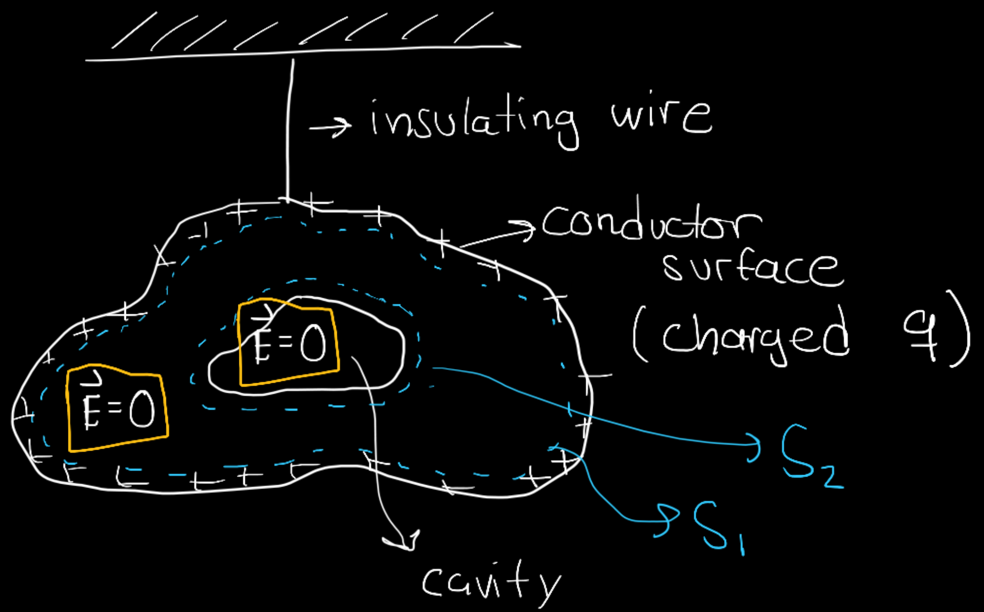
⇒ breaks electrostatic condition

⇒ if $\vec{E} = 0$ then $Q_{enc} = 0$ by Gauss' law $\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

(no charges inside the conductor, they occupy the outer surface of the conductor)

Gauss' law (Chapter 6.4)

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$



→ Is there excess charge in the inner surface of the conductor?

No, charges stay in the outer surface

⇒ the conductor is a "pathway" for the charges to distribute along the surface of the conductor